Math 112

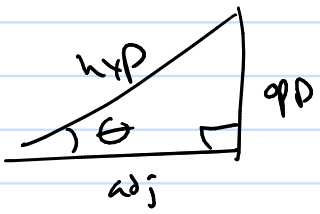
Table

θ	$\cos\theta$	$\sin\theta$	$\tan\theta = \frac{\sin\theta}{\cos\theta}$	$\cot\theta = \frac{\cos\theta}{\sin\theta}$	$\sec\theta = \frac{1}{\cos\theta}$	$\csc\theta = \frac{1}{\sin\theta}$
$0°$ 0	1	0	0	undefined $\cdots \infty$	1	undef. $\cdots \infty$
$30°$ $\pi/6$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}/3$	$\sqrt{3}$	$\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$	2
$45°$ $\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1	1	$\sqrt{2}$	$\sqrt{2}$
$60°$ $\pi/3$	$1/2$	$\sqrt{3}/2$	$\sqrt{3}$	$\sqrt{3}/3$	2	$\frac{2\sqrt{3}}{3}$
$90°$ $\pi/2$	0	1	undefined $\cdots \infty$	0	undefined $\cdots \infty$	1

for $\tan\theta$ [$\cos\theta = 0$ means vertical asym at $\theta = \frac{\pi}{2} + n\pi$, n is any integer

10.4 More Identities.

up to now



$\sin\theta = \frac{opp}{hyp}$ etc.

b/c $adj^2 + opp^2 = hyp^2$

$\sin^2\theta + \cos^2\theta = 1$

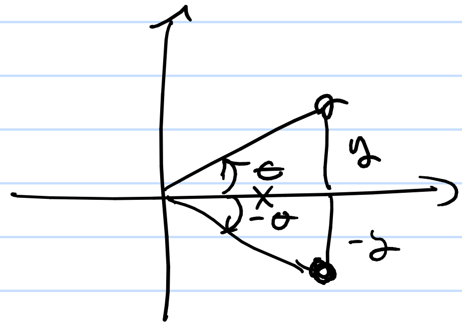
$1 + \cot^2\theta = \csc^2\theta$

etc

More ↻ ↺

Even functions (Sym. about y-axis)

$$f(-x) = f(x)$$



$$\cos(-\theta) = \cos(\theta)$$

$$\sec(-\theta) = \sec(\theta)$$

Odd functions (Sym. about origin)

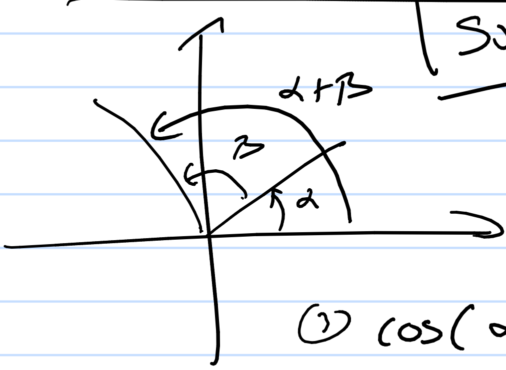
$$f(-x) = -f(x)$$

b/c $\tan\theta = \frac{\sin\theta}{\cos\theta} = \sin\theta\sec\theta$

$$\sin(-\theta) = -\sin(\theta) \qquad \tan(-\theta) = -\tan(\theta)$$

$$\csc(-\theta) = -\csc(\theta) \qquad \cot(-\theta) = -\cot(\theta)$$

Sum / Difference



① $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

② $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

③ $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

④ $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \underbrace{\cos\left(\frac{\pi}{2}\right)}_{=0}\cos(\theta) + \underbrace{\sin\left(\frac{\pi}{2}\right)}_{=1}\sin(\theta) = \sin(\theta)$$

$$\sin\left(\frac{\pi}{2} - \theta\right) = \underbrace{\sin\left(\frac{\pi}{2}\right)}_{=1}\cos(\theta) - \underbrace{\cos\left(\frac{\pi}{2}\right)}_{=0}\sin(\theta) = \cos(\theta)$$

$$\cot\left(\frac{\pi}{2} - \theta\right) = \frac{\cos\left(\frac{\pi}{2} - \theta\right)}{\sin\left(\frac{\pi}{2} - \theta\right)} = \frac{\sin\theta}{\cos\theta} = \tan(\theta)$$

Cofunction Identities

$\cos\left(\frac{\pi}{2} - \theta\right) = \sin(\theta)$ $\quad$ $\sin\left(\frac{\pi}{2} - \theta\right) = \cos(\theta)$

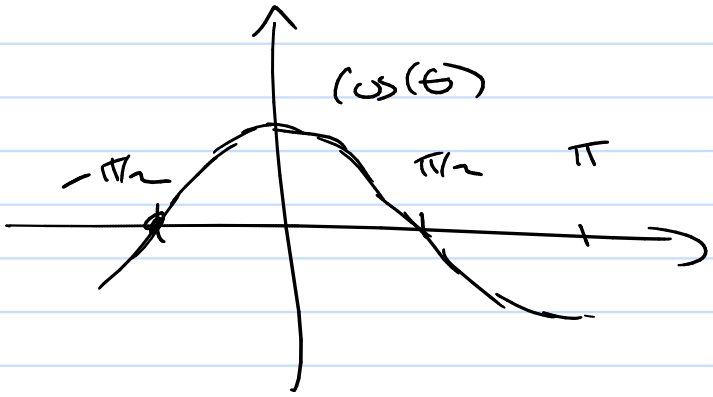
$\tan\left(\frac{\pi}{2} - \theta\right) = \cot(\theta)$ $\quad$ $\cot\left(\frac{\pi}{2} - \theta\right) = \tan(\theta)$

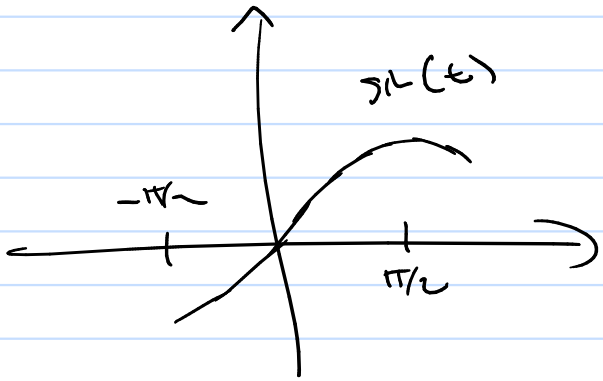
$\sec(\pi/2 - \theta) = \csc(\theta)$ $\quad$ $\csc\left(\frac{\pi}{2} - \theta\right) = \sec(\theta)$

Mix it up

$\sin(\theta) = \cos\left(\frac{\pi}{2} - \theta\right) = \cos\left(-\left(\left(\theta - \pi/2\right)\right)\right)$

$= \cos(\theta - \pi/2)$





Consider $\sin(2\theta) = \sin(\theta + \theta)$

$= \sin\theta\cos\theta + \cos\theta\sin\theta$

Double Angle Formula $\sin(2\theta) = 2\sin\theta\cos\theta$

(★) $\cos(2\theta) = \cos^2\theta - \sin^2\theta$ $\quad$ also know $\boxed{\cos^2\theta + \sin^2\theta = 1}$

$= 1 - 2\sin^2\theta$

$= 2\cos^2\theta - 1$

b/c $\cos(2\theta) = 1 - 2\sin^2\theta$

$2\sin^2\theta = 1 - \cos(2\theta)$

$$\sin^2\theta = \frac{1 - \cos(2\theta)}{2}$$

similar $\cos^2\theta = \frac{1 + \cos(2\theta)}{2}$