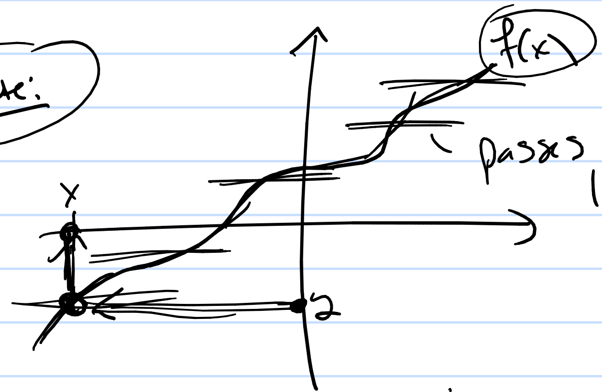
Math 112

Inverse Trig (Circular) functions | 10.6

Note:



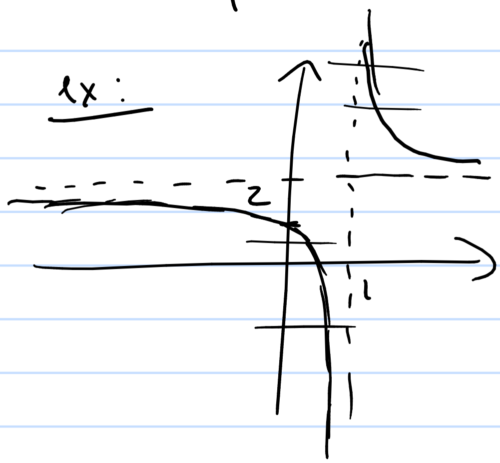
passes horz. line test

there fore $f^{-1}(x)$ exists

and we find by swapping x, y

and solve for y (if possible)

ex:



$y = \frac{1}{x-1} + 2$

f^{-1}?

$x = \frac{1}{y-1} + 2$

solve for y?

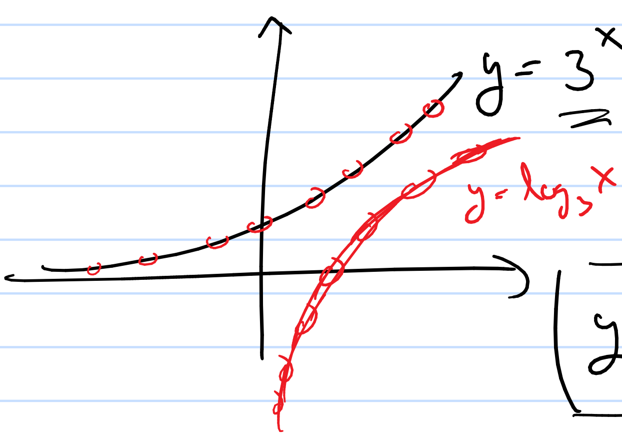
$x - 2 = \frac{1}{y-1}$

or $y - 1 = \frac{1}{x-2}$

$y = \boxed{\frac{1}{x-2} + 1 = f^{-1}(x)}$

We can't always solve for y.

(exponential, logs)



f^{-1} exists

$\rightarrow$ swap x, y

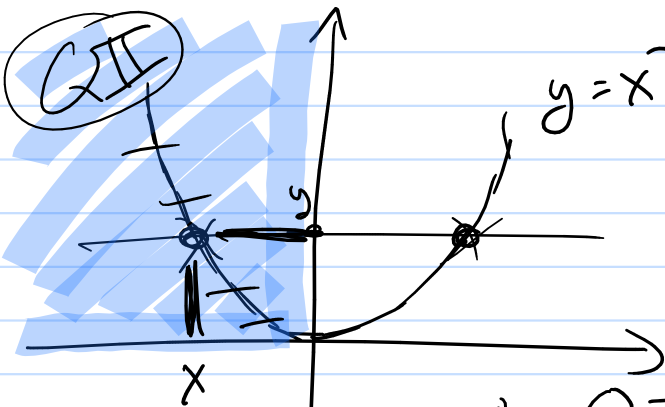
$y = f^{-1}(x)$

$\boxed{x = 3^y}$

define the inverse to be

$\boxed{y = \log_3(x) \text{ means } 3^y = x}$

Don't pass horz. line test?



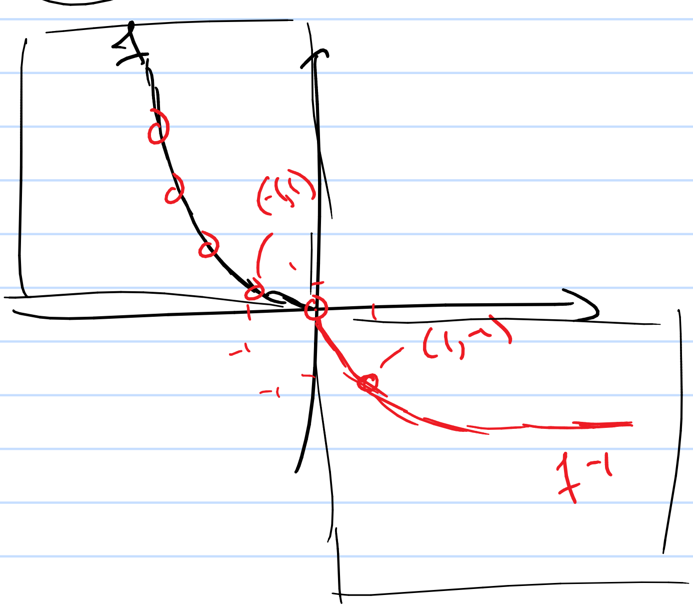
Make a "window" of domain and codomain such that in that "window" it does pass horz. line test

in QII, restrict $\boxed{x \le 0}$ f^{-1} exist.

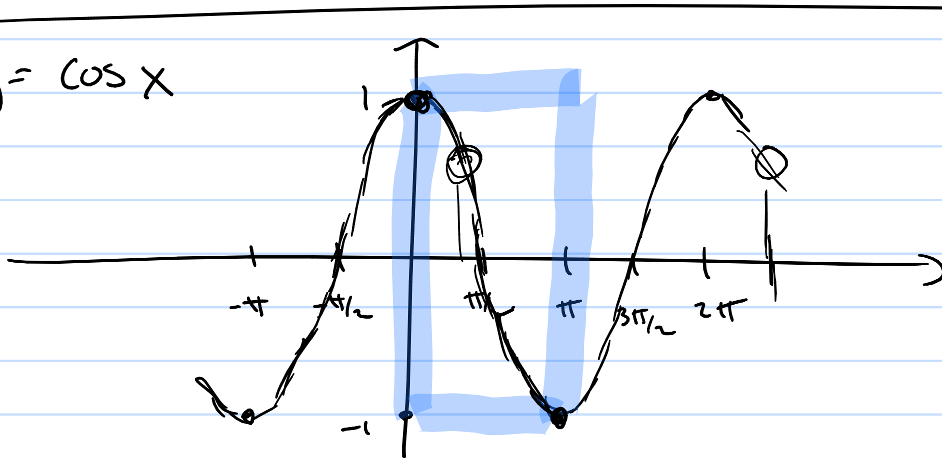
$y = x^2$, $x \le 0$ → $\boxed{\text{Swap } x, y}$ → $x = y^2 \rightarrow$ $y = \sqrt{x}$ or $\boxed{y = -\sqrt{x}}$

$y \le 0$

$$\boxed{f^{-1}(x) = -\sqrt{x}}$$



$y = \cos x$



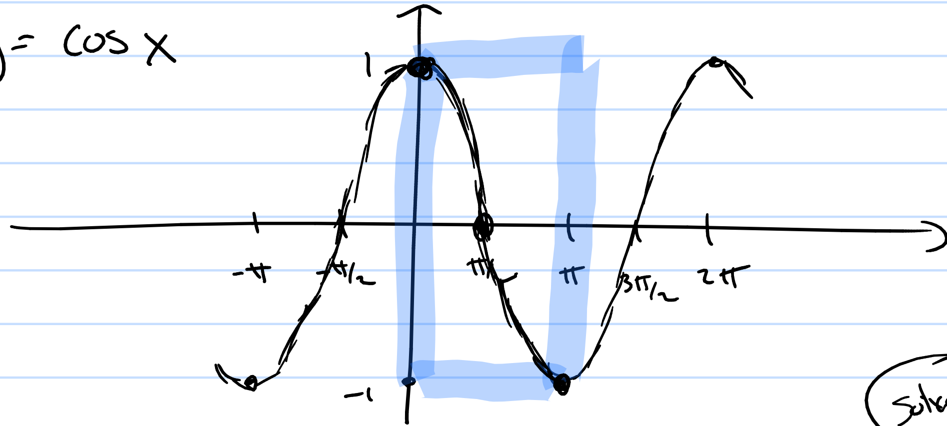
restrict $0 \le x \le \pi$

Range $[-1, 1]$

$y = \cos x$

restrict $0 \le x \le \pi$

Range $[-1, 1]$



find f^{-1} swap x, y $\quad x = \cos y \xrightarrow{\text{solve?}} y = f^{-1}(x)$

x (angle)		$\cos(x)$
0°	0	1
30°	$\pi/6$	$\sqrt{3}/2$
45°	$\pi/4$	$\sqrt{2}/2$
60°	$\pi/3$	$1/2$
90°	$\pi/2$	0
	$2\pi/3$	$-1/2$
	$3\pi/4$	$-\sqrt{2}/2$
	$5\pi/6$	$-\sqrt{3}/2$
	π	-1

x	$\cos^{-1}(x)$
1	0
$\sqrt{3}/2$	$\pi/6$
$\sqrt{2}/2$	$\pi/4$
$1/2$	$\pi/3$
0	$\pi/2$
$-1/2$	$2\pi/3$
$-\sqrt{2}/2$	$3\pi/4$
$-\sqrt{3}/2$	$5\pi/6$
-1	π

↑ ratios ↑ angles

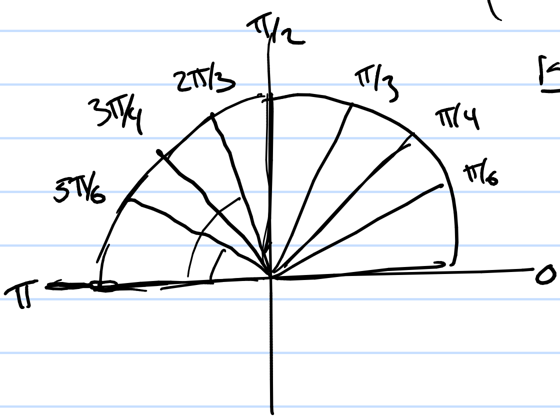
call $y = \text{invcos}(x)$

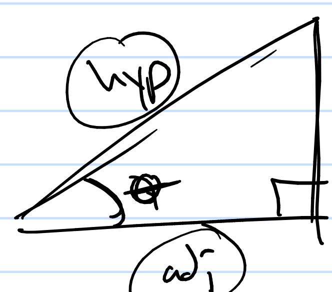
or

$$\boxed{\begin{array}{l} y = \cos^{-1}(x) \quad \leftarrow \text{inv. (not a power)} \\ \text{or} \\ y = \arccos(x) \end{array}}$$

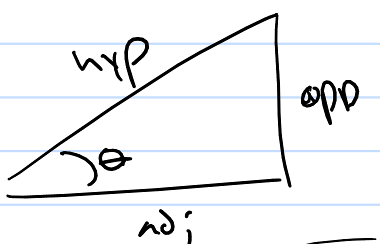
$\cos(\text{angle}) = \text{ratio}$

$\cos^{-1}(\text{ratio}) = \text{angle}$





$$\cos^{-1}\left(\frac{adj}{hyp}\right) = \theta$$



$$\cos(\theta) = \boxed{\frac{adj}{hyp}}$$

out = ratio

in = angle