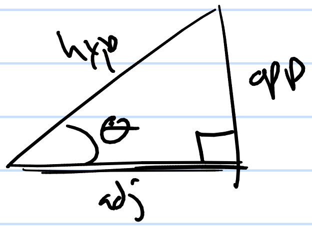
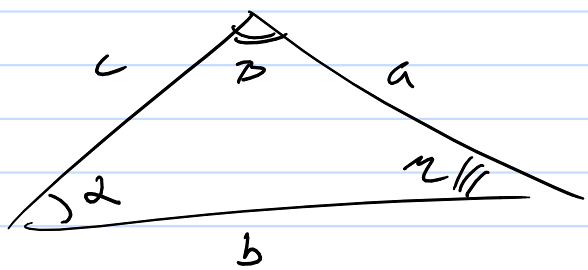
Math 112

Trig:

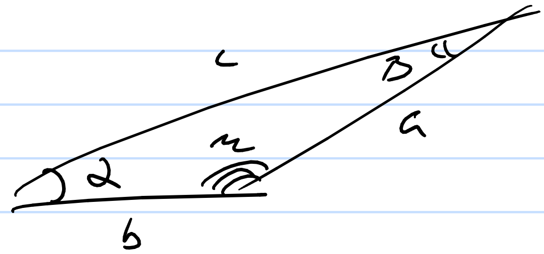


$adj^2 + opp^2 = hyp^2$

$\sin\theta = \frac{opp}{hyp}$ etc etc.



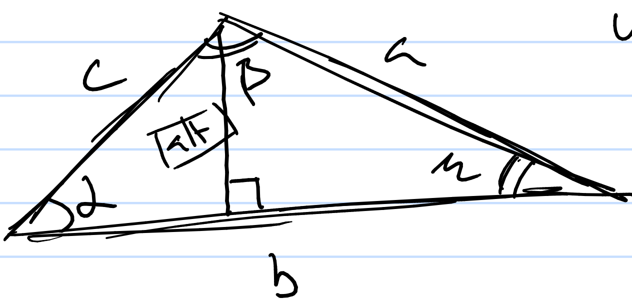
or



etc

Not right triangles.

(no right triangle? Nope)



we can look at this as two right triangles

from this we get

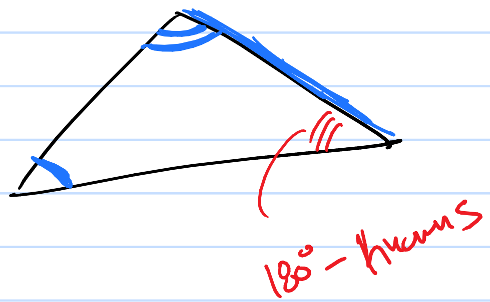
$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

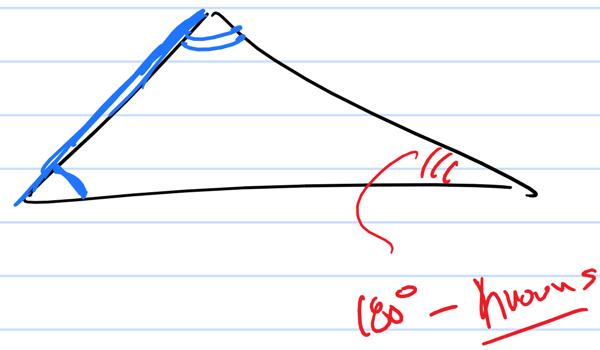
we also know $\alpha + \beta + \gamma = 180°$ ✓

Questions using law of Sines

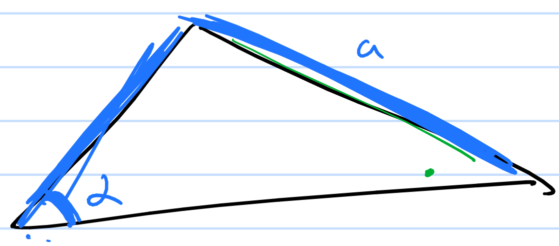
- need two angles → 3rd is easy to find
- with one side we can "solve".

So Angle Angle Side or Angle Side Angle

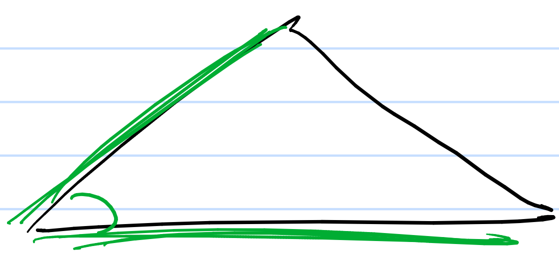




Angle, side, side

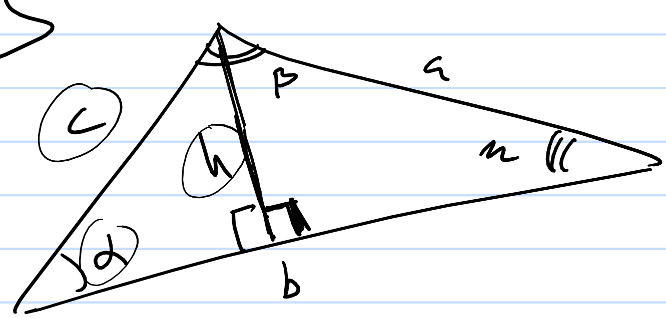


Side Angle Side



Law of Sines does not work!

So



go back to our two right triangle version of the problem.

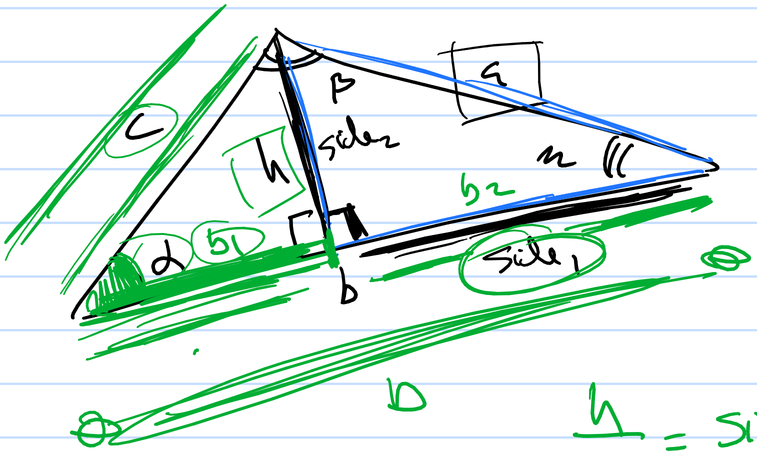
Law of Sines came from $\sin\alpha = \frac{h}{c}$ $\sin\gamma = \frac{h}{a}$

equal!

$c \sin\alpha = h$ $a \sin\gamma = h$

$$c \sin\alpha = a \sin\gamma$$

$$\frac{\sin\alpha}{a} = \frac{\sin\gamma}{c}$$



$$a^2 = \text{side}_1^2 + \text{side}_2^2$$

$$\frac{h}{c} = \sin\alpha$$

$$h = c\sin\alpha$$

$$\frac{b_1}{c} = \cos\alpha$$

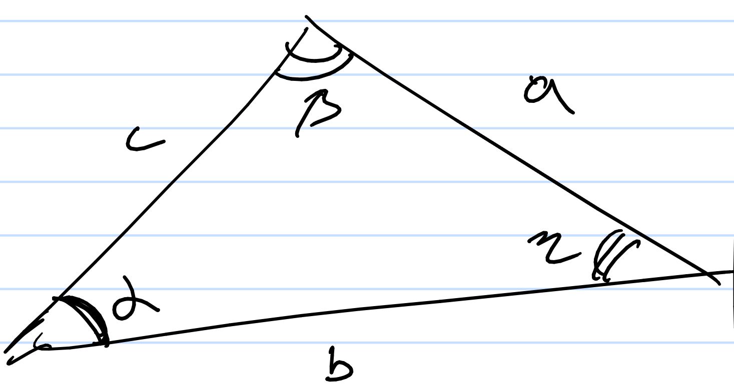
$b_1 = c\cos\alpha$ so $b_2 = b - b_1 = b - c\cos\alpha$

So $a^2 = (b - c\cos\alpha)^2 + (c\sin\alpha)^2$

$$a^2 = b^2 - 2bc\cos\alpha + c^2\cos^2\alpha + c^2\sin^2\alpha$$

$$a^2 = b^2 - 2bc\cos\alpha + c^2[\cos^2\alpha + \sin^2\alpha]$$

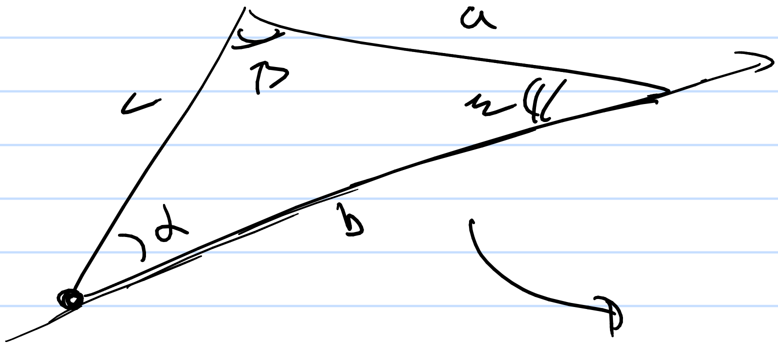
$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$



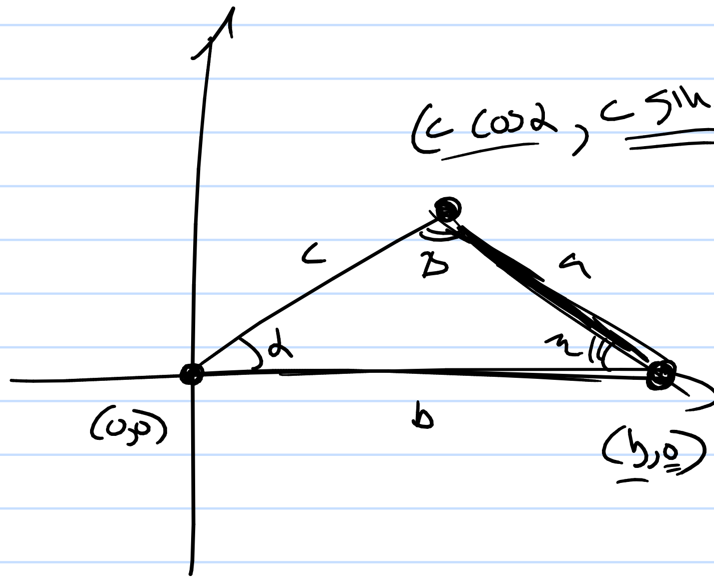
$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$
$$b^2 = a^2 + c^2 - 2ac\cos\beta$$
$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

Law of Cosines

Finding Law of Cosines like the textbook...



choose coord. axis to make the problem easier

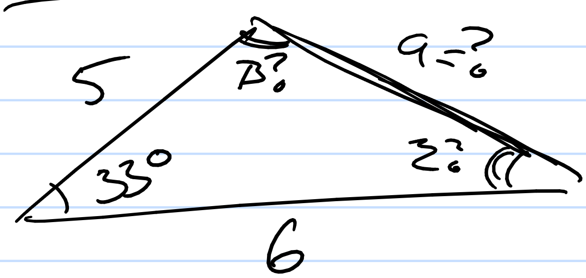


$$a^2 = (c\cos\alpha - b)^2 + (c\sin\alpha - 0)^2$$

$$a^2 = c^2\cos^2\alpha - 2bc\cos\alpha + b^2 + c^2\sin^2\alpha$$

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

Side, Angle, Side



$$a^2 = 5^2 + 6^2 - 2(5)(6)\cos(33^\circ)$$

$$a^2 = 25 + 36 - 60\cos(33^\circ)$$

$$\boxed{a = \sqrt{61 - 60\cos(33^\circ)}}$$

$$\frac{\sin\beta}{6} = \frac{\sin 33^\circ}{\sqrt{61 - 60\cos(33^\circ)}}$$

$$\beta = \sin^{-1}\left(\frac{6\sin(33^\circ)}{\sqrt{61 - 60\cos(33^\circ)}}\right)$$