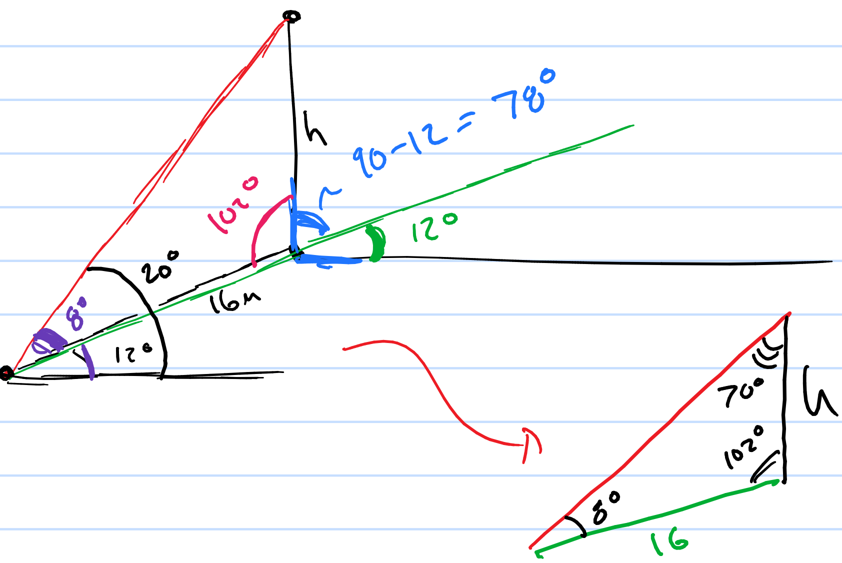
Math 112

Q's

6. A flagpole at right angle to the horizontal is located on a slope that makes an angle of **12°** with the horizontal. The pole's shadow is 16 meters long and points directly *down* the slope. The angle of elevation from the tip of the shadow to the sun is **20°**. The height of the pole is ______ meters.

Hint: Draw a triangle and apply the Law of Sines. Note that the shadow points *down* the slope starting from the bottom of the flag pole. The angle of elevation is the angle made with the horizontal (not with the ground).



$180 - 78 = 102°$

$180 - 8 - 102$

$180 - 110 = 70°$

$$\frac{\sin 8°}{h} = \frac{\sin 70°}{16}$$

$$\boxed{h = 16\,\frac{\sin 8°}{\sin 70°}} = \underline{\text{use calculator}}$$

3 Suppose you are given a triangle with

$$A = 60^\circ, \quad b = 6, \quad c = 9.$$

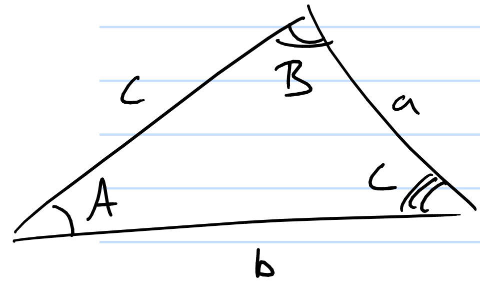
Then

$a =$ [ ] ,

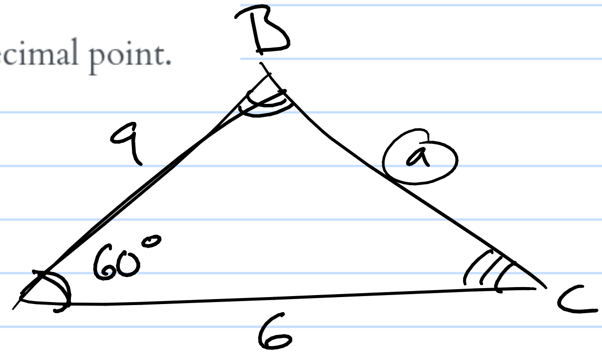
$B =$ [ ] degrees, and

$C =$ [ ] degrees.

Enter your answers with two digits beyond the decimal point.



Tar B



Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$opp^2 = S_1^2 + S_2^2 - 2S_1S_2 \cos\theta$$

$$a^2 = 6^2 + 9^2 - 2\cdot6\cdot9 \cos 60^\circ$$

$$a = \sqrt{6^2 + 9^2 - 2\cdot6\cdot9\cdot\cos(60^\circ)}$$

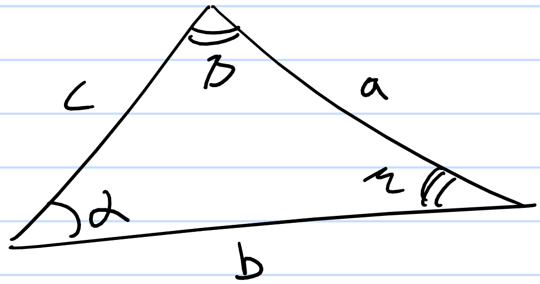
↑ 1/2

θ	$\cos\theta$
0	1
$\pi/6 = 30^\circ$	$\sqrt{3}/2$
$\pi/4 = 45^\circ$	$\sqrt{2}/2$
$\pi/3 = 60^\circ$	$1/2$
$\pi/2 = 90^\circ$	0

$$a = \sqrt{36 + 81 - 54}$$
$$= \sqrt{63}$$

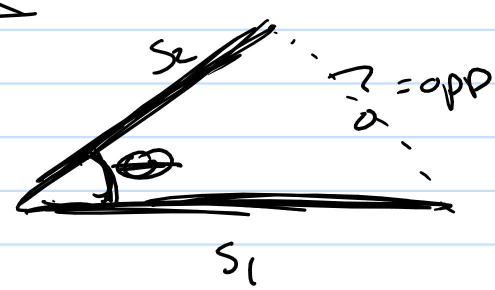
Law of Sines

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$



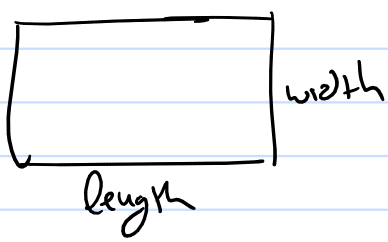
Law of Cosines

$$a^2 = b^2 + c^2 - 2ab\cos\alpha$$



$$opp^2 = s_1^2 + s_2^2 - 2s_1s_2\cos\theta$$

Areas:

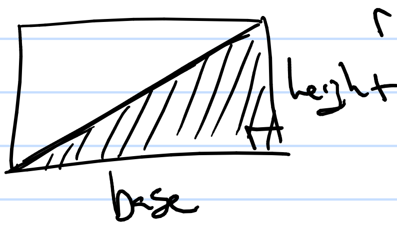


Area = length · width

Perimeter $= 2l + 2w$
$= 2(l+w)$

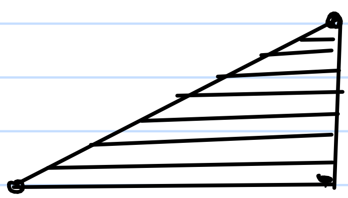
Area $= \pi r^2$ Perimeter $= 2\pi r$

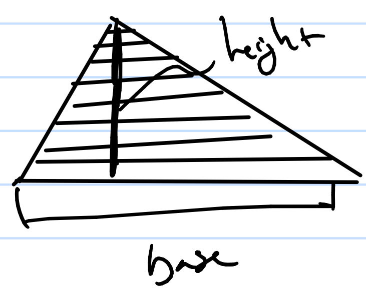
triangle



right triangle $= \frac{1}{2} b h$

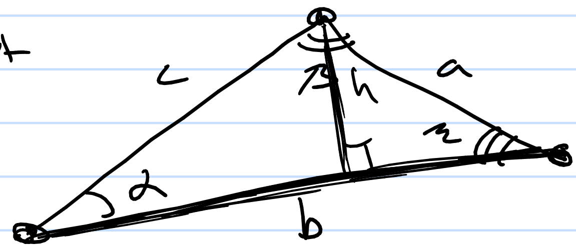
Perimeter $= b + h +$ hypot.





$A = \frac{1}{2} \cdot b \cdot h$

what about



Area?

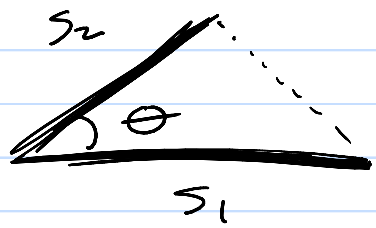
base = b

height = $c \sin\alpha$

$$A = \tfrac{1}{2}\,\text{base}\cdot\text{height} = \tfrac{1}{2}\cdot b\cdot c\sin\alpha$$

$$\boxed{A = \tfrac{1}{2}\, b\cdot c\cdot \sin\alpha}$$

So



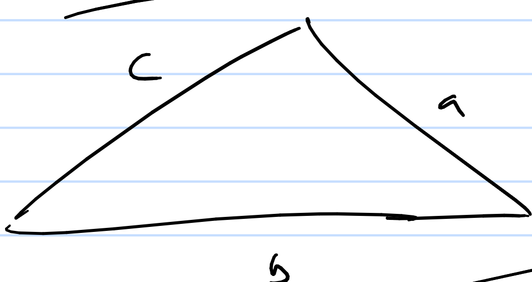
$$\boxed{A = \tfrac{1}{2} s_1 s_2 \sin\theta}$$

from law of cosines

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$2bc\cos\alpha = b^2 + c^2 - a^2$$

$$\cos\alpha = \boxed{\frac{b^2 + c^2 - a^2}{2bc}}$$



A = use only? a's and b's and c's

but $A = \tfrac{1}{2} bc \boxed{\sin\alpha}$

and $\sin^2\alpha + \cos^2\alpha = 1$

$$\sin^2\alpha = 1 - \cos^2\alpha$$

$$\sin\alpha = \sqrt{1 - \boxed{\cos^2\alpha}}$$

$$A = \tfrac{1}{2} bc\sin\alpha = \tfrac{1}{2} bc\sqrt{1 - \cos^2\alpha}$$

$$A = \tfrac{1}{2} bc\sqrt{1 - \left(\frac{b^2 + c^2 - a^2}{2bc}\right)^2}$$

$$A = \frac{1}{2}bc\sqrt{1-\left(\frac{b^2+c^2-a^2}{2bc}\right)^2}$$

$$= \frac{1}{2}bc\sqrt{1-\frac{(b^2+c^2-a^2)^2}{(2bc)^2}} = \frac{1}{2}bc\sqrt{\frac{(2bc)^2-(b^2+c^2-a^2)^2}{(2bc)^2}}$$

$$= \frac{1}{2}\cancel{bc}\,\frac{\sqrt{(2bc)^2-(b^2+c^2-a^2)^2}}{2\cancel{bc}}$$

$$A = \frac{1}{4}\sqrt{(2bc)^2-(b^2+c^2-a^2)^2}$$

$$A = \frac{1}{4}\sqrt{\left((2bc)-(b^2+c^2-a^2)\right)\left((2bc)+(b^2+c^2-a^2)\right)}$$

= more algebra

$$= \sqrt{\underbrace{\boxed{\left(\frac{a+b+c}{2}\right)}}_{= s}\left(\frac{a+b-c}{2}\right)\left(\frac{a-b+c}{2}\right)\left(\frac{-a+b+c}{2}\right)}$$

$$A = \sqrt{s(s-c)(s-b)(s-a)}$$

$$A^2 = s(s-c)(s-b)(s-a) \quad \text{w/ } s = \frac{a+b+c}{2}$$