Math 112

$\boxed{\text{Q's}}$

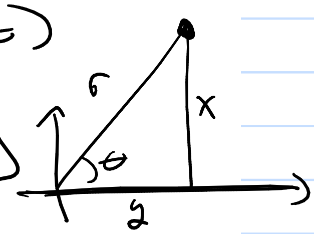
11.4

8 Convert the following rectangular equations into polar equations.

(a) $x^2 + y^2 = 2x$ → $\frac{r^2}{r} = \frac{2(r\cos\theta)}{r}$

$\boxed{r = a\cos(\theta);}$

$a =$ 2 $\quad \boxed{r = 2\cos\theta}$



$x = r\cos\theta$

$y = r\sin\theta$

$x^2 + y^2 = r^2$

$\tan(\theta) = \frac{y}{x}$

(b) $x^2 - y^2 = x$

$r = \frac{\cos(a\theta)}{\cos(b\theta)};$ ← $r = \frac{\cos\theta}{\cos 2\theta}$ (see below)

$a =$ 1 , $b =$ 2 .

(c) $x^4 - y^4 = xy$

$r^2 = a\tan(b\theta);$

$a =$, $b =$.

$$x^2 - y^2 = x$$

$$(r\cos\theta)^2 - (r\sin\theta)^2 = (r\cos\theta)$$

$$r^2\cos^2\theta - r^2\sin^2\theta = r\cos\theta$$

$$r^2\left[\underline{\cos^2\theta - \sin^2\theta}\right] = r\cos\theta$$

$$\frac{r^2}{r}\cos 2\theta = \frac{r\cos\theta}{r}$$

$$\frac{r\cos 2\theta}{\cos 2\theta} = \frac{\cos\theta}{\cos 2\theta} \rightarrow \boxed{r = \frac{\cos\theta}{\cos 2\theta}}$$

(c) $x^4 - y^4 = xy$ $\quad r^2 = \frac{1}{2}\tan(2\theta)$

$r^2 = a\tan(b\theta)$;

$a =$ $\frac{1}{2}$, $b =$ 2 .

but $\quad x^4 - y^4 = (x^2)^2 - (y^2)^2$

$$= (x^2+y^2)(x^2-y^2)$$

So

$$x^4 - y^4 = xy$$

is $\quad (x^2+y^2)(x^2-y^2) = xy$

$$(r^2)(r^2\cos 2\theta) = (r\cos\theta)(r\sin\theta)$$

$$\frac{r^4\cos 2\theta}{r^2} = \frac{r^2\cos\theta\sin\theta}{r^2}$$

$$2r^2\cos 2\theta = 2\cos\theta\sin\theta$$

$$\frac{2r^2\cos 2\theta}{\cos 2\theta} = \frac{\sin 2\theta}{\cos 2\theta}$$

$$\frac{2r^2}{2} = \frac{\tan(2\theta)}{2} \to r^2 = \frac{1}{2}\tan(2\theta)$$

we know $\quad x = r\cos\theta$

$y = r\sin\theta$

$x^2+y^2 = r^2$

from (b) $\to (x^2-y^2) = r^2\cos 2\theta$

and

ans $\Rightarrow r^2 = a\tan(b\theta)$

and $\quad \sin(2\theta) = 2\sin\theta\cos\theta$

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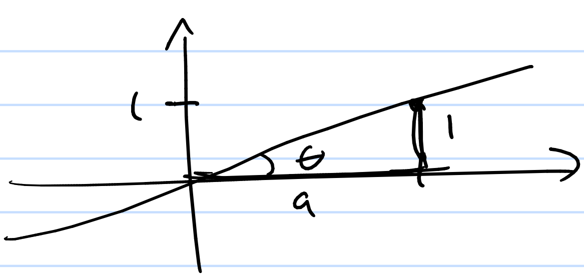
13 Find the equation in polar coordinates of the line through the origin with slope $\frac{1}{9}$.

$\theta =$



$\tan\theta = \frac{1}{9}$

$$\boxed{\theta = \tan^{-1}\left(\frac{1}{9}\right)}$$

(Cont) Systems of Linear Equations

to solve you can find...

exactly one soln

ex $\begin{cases} x+y=2 \\ x-y=0 \end{cases}$

$x+1=2$

$\boxed{x=1}$

step 2

r_1-r_2

$$\begin{aligned} x+y&=2 \\ -(x-y)&=-0 \end{aligned} \rightarrow \begin{aligned} x+y&=2 \\ -x+y&=0 \\ \hline 2y&=2 \\ \boxed{y}&\boxed{=1} \end{aligned}$$

step 1

point $(1,1)$

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no solutions

ex $\begin{cases} x+y=2 \\ x+y=1 \end{cases}$

r_1-r_2

$$\begin{aligned} x+y&=2 \\ -(x+y)&=-(1) \\ \hline 0&=1 \end{aligned}$$

no solution

can not be true

Infinite Solutions

$\begin{cases} x+y=2 \\ 3x+3y=6 \end{cases}$

$3r_1-r_2$

$$\begin{aligned} 3x+3y&=6 \\ -3x-3y&=-6 \\ \hline 0&=0 \end{aligned}$$

always true?

infinite solutions

How to solve:

① Substitution

② Elimination

③ Augmented Matrices

Def a Matrix is a rectangular array of numbers.

ex $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -1 & 0 & \boxed{1} & 2 \end{bmatrix}$ is 2×4 in size

size is rows x columns

we use row, col. to find a location.

ex in the 2,3 place is the number 1.

In general $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & & & & \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$

$m \times n$ size

ex $B = \begin{bmatrix} -1 & 2 & 4 \\ 0 & 1 & 0 \\ -2 & 3 & 7 \\ 0 & 1 & 4 \end{bmatrix}$ size: 4×3

$b_{32} = 3$

$b_{41} = 0$

$b_{11} = -1$

why?

$$x + y = 2$$
$$x - y = 0$$

is positional number

cost. Matrix

$1\ 3\ 7$

Coeff. Matrix $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$

$= 1 \cdot 100 + 3 \cdot 10 + 7 \cdot 1$

Augmented Matrix:

$$\left[\begin{array}{cc|c} 1 & 1 & 2 \\ 1 & -1 & 0 \end{array}\right]$$

$$x + y = 2$$
$$x - y = 0$$

$r_1 - r_2 :\ 0 \cdot x + 2y = 2$

$$\tfrac{1}{2}\, 2y = 2\, \tfrac{1}{2}$$

$$\boxed{y = 1}$$

$$x + y = 2$$

so $x + 1 = 2$

$$\boxed{x = 1}$$

$$\left[\begin{array}{cc|c} 1 & 1 & 2 \\ 1 & -1 & 0 \end{array}\right]$$

$r_1 - r_2$ = New r_2 $\left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 2 & 2 \end{array}\right]$

$\frac{1}{2} r_2$ = New r_2 $\left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 1 & 1 \end{array}\right]$

x's y's = #'s

$$\boxed{y = 1}$$

back solve

$$x + y = 2$$
$$x + 1 = 2$$
$$\boxed{x = 1}$$

Solving Systems of Linear Equations by Aug. Matrices

1. Write your aug. matrix
2. use **row operations**
   - (a) swap rows
   - (b) mult. row by non-zero = New row
   - (c) Add multiples of rows = New row
3. place in row esch. form.
   - (a) back solve
   - or
   - (b) reduced row esch. form and read your answer.

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ex

$$\left[\quad \middle| \quad\right] \xrightarrow[\text{ops}]{\text{row}} \begin{array}{cccc} x & y & z & = \text{const} \\ \end{array}$$

$$\underbrace{\left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 1 & 3 & 7 \\ 0 & 0 & 1 & 2 \end{array}\right]}_{\text{row esch.}} \quad \boxed{z = 2}$$

r_2 is $y + 3z = 7$ but $z = 2$

$y + 3(2) = 7$ so $\boxed{y = 1}$

r_1 is $x + 2y - z = 1$ but $z = 2$, $y = 1$

$x + 2(1) - 2 = 1$ so $\boxed{x = 1}$

ans $\boxed{(1, 1, 2)}$