Math 112

Q's

$x = r\cos\theta$

$y = r\sin\theta$

$r^2 = x^2 + y^2$

$\tan\theta = \frac{y}{x}$

6 A curve with polar equation

1 pt

$$r = \frac{25}{9\sin\theta + 19\cos\theta}$$

$r(9\sin\theta + 19\cos\theta) = 25$

$9r\sin\theta + 19r\cos\theta = 25$

$9y + 19x = 25$

Finish

represents a line. This line has a Cartesian equation of the form $y = mx + b$, where m and b are constants. Give the formula for y in terms of x. For example, if the line had equation $y = 2x + 3$ then the answer would be $2 * x + 3$.

Hint: multiply both sides by the denominator on the right hand side and use $r\cos\theta = x$ and $r\sin\theta = y$.

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7 Solve the system using elimination.

1 pt

$$\begin{cases} -6x + 3y + 3z = 30 \\ 5x - 2y - 6z = -45 \\ 3x + 3y - 4z = -50 \end{cases}$$

$x =$

$y =$

$z =$

elimination

$-\frac{1}{6} r_1 = \text{New } r_1$

$$\begin{cases} x - \frac{1}{2}y - \frac{1}{2}z = -5 \\ 5x - 2y - 6z = -45 \\ 3x + 3y - 4z = -50 \end{cases}$$

$-5r_1 + r_2 = N r_2$

$-3r_1 + r_3 = N r_3$

$$\begin{cases} x - \frac{1}{2}y - \frac{1}{2}z = -5 \\ \frac{1}{2}y - \frac{7}{2}z = -20 \\ \frac{9}{2}y - \frac{5}{2}z = -35 \end{cases}$$

$2r_2 = N r_2$

$2r_3 = N r_3$

$$\left[\begin{array}{ccc|c} -6 & 3 & 3 & 30 \\ 5 & -2 & -6 & -45 \\ 3 & 3 & -4 & -50 \end{array}\right]$$

$-\frac{1}{6} r_1 = N r_1$

Aug. Matrix

$-\frac{1}{6} r_1 = \text{New } r_1$

$$\left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & -\frac{1}{2} & -5 \\ 5 & -2 & -6 & -45 \\ 3 & 3 & -4 & -50 \end{array}\right]$$

-3

$-5r_1 + r_2 = N r_2$

$-3r_1 + r_3 = N r_3$

$15 - 50$

$$\left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & -\frac{1}{2} & -5 \\ 0 & \frac{1}{2} & -\frac{7}{2} & -20 \\ 0 & \frac{9}{2} & -\frac{5}{2} & -35 \end{array}\right]$$

$2r_2 = N r_2$

$2r_3 = N r_3$

$$\begin{cases} x - \frac{1}{2}y - \frac{1}{2}z = -5 \\ \frac{1}{2}y - \frac{7}{2}z = -20 \\ \frac{9}{2}y - \frac{5}{2}z = -35 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & -\frac{1}{2} & -5 \\ 0 & \frac{1}{2} & -\frac{7}{2} & -20 \\ 0 & \frac{9}{2} & -\frac{5}{2} & -35 \end{array}\right]$$

$2r_2 = Nr_2$

$2r_3 = Nr_3$

$2r_2 = Nr_2$

$2r_3 = Nr_3$

$$\begin{cases} x - \frac{1}{2}y - \frac{1}{2}z = -5 \\ y - 7z = -40 \\ 9y - 5z = -70 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & -\frac{1}{2} & -5 \\ 0 & 1 & -7 & -40 \\ 0 & 9 & -5 & -70 \end{array}\right] \leftarrow (-9)$$

$-9r_2 + r_3 = Nr_3$

$-9r_2 + r_3 = Nr_3$

$$\begin{cases} x - \frac{1}{2}y - \frac{1}{2}z = -5 \\ y - 7z = -40 \\ 58z = 290 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & -\frac{1}{2} & -5 \\ 0 & 1 & -7 & -40 \\ 0 & 0 & 58 & 290 \end{array}\right]$$

$\frac{1}{58} r_3 = Nr_3$

$\frac{1}{58} r_3 = Nr_3$

$$\begin{cases} x - \frac{1}{2}y - \frac{1}{2}z = -5 \\ y - 7z = -40 \\ z = 5 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & -\frac{1}{2} & -5 \\ 0 & 1 & -7 & -40 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$$\begin{bmatrix} x - \frac{1}{2}y - \frac{1}{2}z = -5 \\ y - 7z = -40 \\ z = 5 \end{bmatrix}$$

backsolve

$$\boxed{\begin{array}{l} x = -5. \\ y = -5. \\ z = 5. \end{array}}$$

$$\left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & -\frac{1}{2} & -5 \\ 0 & 1 & -7 & -40 \\ 0 & 0 & \boxed{1} & 5 \end{array}\right] \leftarrow \frac{1}{2}$$

$\frac{1}{2}r_3 + r_1 = Nr_1$

$7r_3 + r_2 = Nr_2$

$$\left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & 0 & -\frac{5}{2} \\ 0 & \boxed{1} & 0 & -5 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & -\frac{1}{2} & -5 \\ 0 & 1 & -7 & -40 \\ 0 & 0 & \boxed{1} & 5 \end{array}\right] \leftarrow \frac{1}{2}$$

$\frac{1}{2} r_3 + r_1 = N r_1$

$7 r_3 + r_2 = N r_2$

$$\left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & 0 & -\frac{5}{2} \\ 0 & \boxed{1} & 0 & -5 \\ 0 & 0 & 1 & 5 \end{array}\right] \leftarrow \left(\frac{1}{2}\right)$$

$\frac{1}{2} r_2 + r_1 = N r_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$-\frac{5}{2} - \frac{5}{2}$

$x = -5$
$y = -5$
$z = 5$

Introduced Matrices to Solve Systems of Linear Eqn's

Math = toys + rules

rules → $=?$

→ addition?

→ "Multiplication"?

→ Transpose

toys → Matrix of $m \times n$ size

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

① **Equality**

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \text{ vs } \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

$\neq$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \text{ vs } \begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix}$$

$\neq$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

yes

$\underline{A = B}$ if

$A = [a_{ij}]$ $B = [b_{ij}]$

① Same size

② for all i, j locations $a_{ij} = b_{ij}$

② **Addition** $A + B = [a_{ij} + b_{ij}]$ with both A, B being $\underline{m \times n}$

same size

$$(x + 3y) + (-2x + y) = -x + 4y$$

$$[1 \ 3] + [-2 \ 1] = [1 + -2 \quad 3 + 1]$$

$$= [-1 \quad 4]$$

ex

$$\begin{bmatrix} 1 & -1 & 0 \\ 3 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 4 \\ -1 & -1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 4 \\ 2 & 0 & 0 \end{bmatrix}$$