

Math 112

Q's

12

Susan places \$5600 in three investments at rates of 10%, 12% and 18% per annum, respectively. The total income after one year is \$ 826.00. If the amount placed in the third investment is \$1200 more than the amount placed in the second, find the amount of each investment.

Your answer is:

Amount at 10% equals: \$

Amount at 12% equals: \$

Amount at 18% equals: \$

Unknowns:

Invest #1
 A_1

Invest #2
 A_2

Invest #3
 A_3

$$\textcircled{1} \quad A_1 + A_2 + A_3 = 5600$$

$$\textcircled{2} \quad \underline{A_3 = A_2 + 1200}$$

$$\textcircled{3} \quad .1 A_1 + .12 A_2 + .18 A_3 = 826$$

System:

$$\begin{aligned} A_1 + A_2 + A_3 &= 5600 \\ -A_2 + A_3 &= 1200 \\ .1 A_1 + .12 A_2 + .18 A_3 &= 826 \end{aligned}$$

or

$$\begin{bmatrix} x + y + z = 5600 \\ -y + z = 1200 \\ .1x + .12y + .18z = 826 \end{bmatrix}$$

as Augmented Matrix

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 5600 \\ 0 & -1 & 1 & 1200 \\ .1 & .12 & .18 & 826 \end{array} \right]$$

Solve

1 Convert the augmented matrix

$$\begin{array}{c} \text{coeff} \\ \downarrow \\ \begin{array}{c|c} 0 & -5 \\ -3 & 3 \\ 1 & 5 \end{array} \\ \downarrow \\ \text{const} \end{array}$$

typical calculator

$$[[0, -5], [-3, 3], [1, 5]]$$

to the equivalent linear system. Use x1 to enter the variable x1.

$$\begin{array}{l} 0 \cdot x_1 = -5 \\ -3 \cdot x_1 = 3 \\ 1 \cdot x_1 = 5 \end{array}$$

$$[0, -5; -3, 3; 1, 5]$$

Matlab or Octave

$$\begin{array}{l} -5x_1 + 4x_2 - 37x_3 = 37 \\ -x_1 + x_2 - 8x_3 = 8 \end{array}$$

$$\left[\begin{array}{ccc|c} -5 & 4 & -37 & 37 \\ -1 & 1 & -8 & 8 \end{array} \right]$$

to an augmented matrix. Then transform the system to reduced echelon form and determine if the system is consistent. If the system is consistent, then find all solutions.

Augmented matrix:

Reduced echelon form:

Is the system consistent?

Solution: $(x_1, x_2) = (\text{input} + \text{input} s_1, \text{input} + \text{input} s_1)$

Swap r1, r2 and take neg

$$\left[\begin{array}{ccc|c} 1 & -1 & 8 & -8 \\ -5 & 4 & -37 & 37 \end{array} \right] \quad (5)$$

$5r_1 + r_2 = Nr_2$

$$\left[\begin{array}{ccc|c} 1 & -1 & 8 & -8 \\ 0 & -1 & 3 & -3 \end{array} \right]$$

$-r_2 = Nr_2$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 8 & -8 \\ 0 & 1 & -3 & 3 \end{array} \right]$$

row esch.

Note:

row esch.

$$\text{(ex)} \rightarrow \left[\begin{array}{ccccc|c} 1 & 1 & -1 & 0 & 2 & 3 \\ 0 & 0 & 1 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 & 2 & 4 \end{array} \right]$$

if a variable is at a lead it is free to take a any value.

(cont)

$$5r_1 + r_2 = Nr_2$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 8 & -8 \\ 0 & -1 & 3 & -3 \end{array} \right]$$

$$-r_2 = Nr_2$$

$$\rightarrow \begin{array}{c} x \quad y \quad z \text{ free} \\ \left[\begin{array}{ccc|c} 1 & -1 & 8 & -8 \\ 0 & 1 & -3 & 3 \end{array} \right] \end{array} \quad \text{row esch.}$$

reduced row esch. Make zeros above leads

$$r_1 + r_2 = Nr_1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 5 & -5 \\ 0 & 1 & -3 & 3 \end{array} \right]$$

is reduced row esch.

Input into @foxy $\left[[1, 0, 5, -5], [0, 1, -3, 3] \right]$

$$\begin{array}{l} -5x_1 + 4x_2 - 37x_3 = 37 \\ -x_1 + x_2 - 8x_3 = 8 \end{array}$$

to an augmented matrix. Then transform the system to reduced echelon form and determine if the system is consistent. If the system is consistent, then find all solutions.

Augmented matrix:

Reduced echelon form:

Is the system consistent?

Solution: $(x_1, x_2) = \left(\begin{array}{c} -5 \\ 3 \\ 0 \end{array} + \begin{array}{c} -5 \\ 3 \\ 1 \end{array} s_1 \right)$

inconsistent \equiv no answers

$$\text{ex } \left[\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & 0 & 4 \end{array} \right]$$

no soln. b/c $0 \neq 4$
call it inconsistent.

$$\rightarrow \begin{array}{c} x \quad x_2 \quad x_3 \text{ free} \\ \left[\begin{array}{ccc|c} 1 & 0 & 5 & -5 \\ 0 & 1 & -3 & 3 \end{array} \right] \end{array}$$

$$\rightarrow \begin{array}{l} x_1 + 5x_3 = -5 \\ x_2 - 3x_3 = 3 \end{array}$$

but x_3 is free

$$x_3 = s_1 \text{ is any real num.}$$

$$\begin{array}{l} x_2 = 3 + 3s_1 \\ x_1 = -5 - 5s_1 \end{array}$$

Ch 8

we will do for exam 8.1 to 8.4

Skills? ① system \rightarrow solve it viz sub. & elim.

② system \rightarrow Aug Matrix \rightarrow solve it using row operations

Matrix Arithmetic

A, B are matrices

① $A + B = [a_{ij} + b_{ij}]$ if A, B are the same size

ex
$$\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 1 & 3 \end{bmatrix} + \begin{bmatrix} -1 & -1 \\ 2 & 1 \\ 3 & -3 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 2 & 3 \\ 4 & 0 \end{bmatrix}$$

② Scalar Multiplication (Distribution)

$$3(2x - y) = 6x - 3y$$

α is a real number

$$\alpha A = [\alpha a_{ij}]$$

ex
$$2 \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 0 & 4 \\ 2 & 6 \end{bmatrix}$$

③ Subtraction

$$A - B = A + (-1)B$$

$$= [a_{ij}] + [-b_{ij}]$$

$$= [a_{ij} - b_{ij}]$$

ex

$$\begin{bmatrix} 1 & -1 \\ 2 & 0 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} -1 & 2 \\ 1 & -1 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ 1 & 1 \\ 0 & 4 \end{bmatrix}$$

④ Uniq. to matrices

Transpose?

Swap
Rvs and
Columns

$$\begin{bmatrix} 1 & -1 \\ 2 & 3 \\ 1 & 0 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 3 & 0 \end{bmatrix}$$

⑤ Matrix • Matrix

(Matrix Multiply)