Math 112

Matrix · Matrix?

Def: ① Matrix $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & & & \\ a_{m1} & \cdots & & a_{mn} \end{bmatrix} = [a_{ij}]$

size $m \times n$ (rows × cols)

② if a matrix is $1 \times n$ call it **row vector**

ex $[1 \ -2 \ 0 \ 3]$

representing a row vector is $\mathbf{v} = [v_1 \ v_2 \cdots v_n]$ ← bold font

→ w/ pencil $\vec{V} = [v_1 \ v_2 \cdots v_n]$

③ if a matrix is $m \times 1$ call it a **column vector**

ex $\vec{V} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_m \end{bmatrix}$

why? $3x + 2y - z = 2 \rightarrow [3 \ 2 \ -1] \begin{bmatrix} x \\ y \\ z \end{bmatrix} = [2]$

1×3, 3×1, 1×1

coeff: $[3 \ 2 \ -1]$

Variables: $x, y, z \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

Def Scalar (or inner) product $[x_1 \ x_2 \ x_3 \ x_4] \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = x_1 y_1 + x_2 y_2 + x_3 y_3 + x_4 y_4$

$$\begin{bmatrix} 1 & 0 & -1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ -3 \\ 2 \\ 1 \end{bmatrix} = 1\cdot 2 + 0\cdot 1 + (-1)(-3) + 2\cdot 2 + (3)(1)$$

$$= \boxed{12}$$

Matrix · Matrix

$$A \quad B = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1k} \\ & & & \\ a_{m1} & \cdots & & a_{mk} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & & & \\ \vdots & & & \\ b_{k1} & & & b_{kn} \end{bmatrix}$$

$m \times k \quad k \times n$

↑ row ↑ col ↑

$m \times k$ $\qquad$ $k \times n$

A's row 1 inner prod w/ A's col 1

$$AB = C = \begin{bmatrix} c_{11} & & \\ & & \\ & & \end{bmatrix}$$

$m \times n$

ex

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 12 & 13 \\ \square & \square & \square \end{bmatrix}$$

$2 \times 2 \qquad 2 \times 3 \qquad 2 \times 3$

$$= \begin{bmatrix} -1 & 5 & 2 \\ -3 & 11 & 4 \end{bmatrix}$$

$$A = \begin{bmatrix} 5 & -5 \\ -3 & 3 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 & 5 \\ -5 & -4 \\ 2 & -5 \end{bmatrix},$$

$$C = \begin{bmatrix} -1 & 1 & -3 \\ 4 & -4 & -5 \end{bmatrix}.$$

If possible, compute the following. If an answer does not exist, enter **DNE**.

$CB - A =$

$$\begin{bmatrix} -1 & 1 & -3 \\ 4 & -4 & -5 \end{bmatrix} \begin{bmatrix} 1 & 5 \\ -5 & -4 \\ 2 & -5 \end{bmatrix} - \begin{bmatrix} 5 & -5 \\ -3 & 3 \end{bmatrix} = \begin{bmatrix} -12 & 6 \\ 14 & 61 \end{bmatrix} - \begin{bmatrix} 5 & -5 \\ -3 & 3 \end{bmatrix}$$

$2 \times 3 \quad 3 \times 2$

$$= \begin{bmatrix} -17 & 11 \\ 17 & 58 \end{bmatrix}$$

$CA =$

Linear Algebra Arithmetic

1. $A + B$
2. αA
3. $A - B$
4. AB

Matrix Multiple is **not** commutative

Note: consider

$$\begin{bmatrix} 1 & 2 & 3 \\ -1 & 4 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1 & 1 & 1 \\ 0 & 1 & 2 \\ 1 & 4 & 2 \end{bmatrix}$$

$2 \times 3 \quad 3 \times 3$

$$= \begin{bmatrix} \; \end{bmatrix}$$

2×3

vs

$$\begin{bmatrix} -1 & 1 & 1 \\ 0 & 1 & 2 \\ 1 & 4 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ -1 & 4 & 1 \end{bmatrix} = \text{dne}$$

$3 \times 3 \quad 2 \times 3$

$3 \neq 2$

(wrong sizes)

Algebra

A is Arithmetic w/o unknowns

college Algebra on real numbers

$3x + 4 = 1$ solve

$3x + 4 - 4 = 1 - 4$

$3x + 0 = 1 - 4$

$\frac{1}{3}(3x) = \frac{1}{3}(-3)$

$1 \cdot x = -1$

$x = -1$

Addition: Identity is 0

$x + 0 = x$

Inverse of a number is $n + (-n) = 0$

(0 ↑ Identity)

Multiplication:

Identity is 1

$x \cdot 1 = x$

Inverse of a number is $n \cdot (n^{-1}) = 1$

Matrix Algebra

$$\begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix} X + \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

(ex)

$$\underset{2 \times 3}{\begin{bmatrix} 1 & 2 & 1 \\ -1 & 3 & 1 \end{bmatrix}} + \underset{2 \times 3}{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}} = \underset{2 \times 3}{\begin{bmatrix} 1 & 2 & 1 \\ -1 & 3 & 1 \end{bmatrix}}$$

Matrix Addition

① Identity is O matrix which is a matrix of all zeros

② Additive inv. of $A = [a_{ij}]$ is $(-1)A$

b/c $A + (-1)A = O_{n \times n}$

Multiplication

① Identity $I_{n \times n} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

② Inverse

ex

$$\underset{2 \times 3}{\begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \end{bmatrix}} \cdot \underset{3 \times 3}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \end{bmatrix}$$

$$\underset{2 \times 2}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}} \cdot \underset{2 \times 3}{\begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \end{bmatrix}} = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \end{bmatrix}$$