Math 112

Q's

Let

$$A = \begin{bmatrix} -1 & -3 \\ -1 & 5 \end{bmatrix},$$

$$B = \begin{bmatrix} 4 & 5 \\ 3 & 3 \\ -2 & 3 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 2 & 5 \\ -3 & 4 & -4 \end{bmatrix}.$$

$$CB = \begin{bmatrix} 1 & 2 & 5 \\ -3 & 4 & -4 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 3 & 3 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 26 \\ 8 & -15 \end{bmatrix}$$

2×3 $\quad 3 \times 2$ $\quad 2 \times 2$

If possible, compute the following. If an answer does not exist, enter **DNE**.

$CB - A =$

$$\begin{bmatrix} 0 & 26 \\ 8 & -15 \end{bmatrix} - \begin{bmatrix} -1 & -3 \\ -1 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 29 \\ 9 & -20 \end{bmatrix} = [[1,29],[9,-20]]$$

help (matrices)

$CA =$

8.2 #8

$$25x_1 + 5x_2 + 5x_3 = 10$$
$$5x_1 + 1x_2 + 1x_3 = 2$$
$$15x_1 + 3x_2 + 3x_3 = 6$$

$$\left[\begin{array}{ccc|c} 25 & 5 & 5 & 10 \\ 5 & 1 & 1 & 2 \\ 15 & 3 & 3 & 6 \end{array}\right]$$

$\frac{1}{25} r_1 = N r_1$ $\rightarrow$

$$\left[\begin{array}{ccc|c} \boxed{1} & 1/5 & 1/5 & 2/5 \\ 5 & 1 & 1 & 2 \\ 15 & 3 & 3 & 6 \end{array}\right]$$

$-5r_1 + r_2 = N r_2$

$-15 r_1 + r_3 = N r_3$

$\rightarrow$

$$\left[\begin{array}{ccc|c} \boxed{1} & 1/5 & 1/5 & 2/5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 \quad x_2 \quad x_3$

x_2 ↑ free, x_3 ↑ free

any real numbers

$$\boxed{x_2 = a_1}$$
$$\boxed{x_3 = a_2}$$

1)

$$x_1 + \frac{1}{5}x_2 + \frac{1}{5}x_3 = 4/5$$
$$x_1 + \frac{1}{5}a_1 + \frac{1}{5}a_2 = 4/5$$
$$\boxed{x_1 = 4/5 - \frac{1}{5}a_1 - \frac{1}{5}a_2}$$

ex

Note: & lead vs free

$$
\begin{array}{cccc|c}
x_1 & x_2 & x_3 & x_4 & \\
\boxed{1} & 2 & -4 & 1 & 0 \\
0 & 0 & \boxed{1} & 2 & 4 \\
0 & 0 & 0 & 0 & 0 \\
\text{lead} & \text{free} & \text{lead} & \text{free} &
\end{array}
$$

free ← b/c no lead in the col.

$4R_2 + R_1 \to R_1$

$$
\left[\begin{array}{cccc|c}
\boxed{1} & 2 & 0 & 9 & 16 \\
0 & 0 & \boxed{1} & 2 & 4 \\
0 & 0 & 0 & 0 & 0
\end{array}\right]
$$

if x_2 is free, then $x_2 =$ any real number

x_4 is free, then $x_4 =$ any real number

---

ex

$$
\left[\begin{array}{ccc|c}
\boxed{1} & 0 & 0 & -2 \\
0 & \boxed{1} & 0 & 3 \\
0 & 0 & 0 & 4
\end{array}\right]
\begin{array}{l}
\leftarrow x = -2 \\
\leftarrow y = 3 \\
\leftarrow 0 = 4 \;\; ?!
\end{array}
$$

$x \quad y \quad z$

o o

No soln

Matrix-Matrix Multiplication

Identity is $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

given a matrix, A, what is the matrix such that

$$A \cdot A^{-1} = I$$

and

$$A^{-1} \cdot A = I$$

call the matrix to be A's inverse

<u>and</u> A is the inverse of that matrix.

<u>Note:</u> we can verify that two matrices M_1, M_2 are inverses by multiplying them.

① $M_1 \cdot M_2 = I$

② $M_2 \cdot M_1 = I$

<u>show both</u>? they are <u>inverses</u>

$\boxed{Q}$ given A ...

① does A^{-1} exist at all?

1st ② How to find A^{-1}?

create an augmented matrix of $[A \mid I]$

$$[A \mid I] \xrightarrow{\text{row ops}} [I \mid A^{-1}]$$

ex find A^{-1} for $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

$$\left[\begin{array}{cc|cc} 2 & 3 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{array}\right] \xrightarrow{\frac{1}{2}r_1 = Nr_1} \left[\begin{array}{cc|cc} \boxed{1} & 3/2 & 1/2 & 0 \\ 1 & 2 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{(-1)r_1 + r_2 = Nr_2} \left[\begin{array}{cc|cc} 1 & 3/2 & 1/2 & 0 \\ 0 & 1/2 & -1/2 & 1 \end{array}\right] \xrightarrow{2r_2 = Nr_2} \left[\begin{array}{cc|cc} \boxed{1} & 3/2 & 1/2 & 0 \\ 0 & \boxed{1} & -1 & 2 \end{array}\right] (-3/2)$$

$$\xrightarrow{-\frac{3}{2}r_2 + r_1 = Nr_1} \left[\begin{array}{cc|cc} 1 & 0 & 2 & -3 \\ 0 & 1 & -1 & 2 \end{array}\right]$$

$$\left[\, I \mid A^{-1} \,\right]$$

check: ① $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ✓

② $\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ✓

Uses? ex $X\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 2 & -1 \end{bmatrix}$

$$X\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 5 \\ 2 & 0 \end{bmatrix}$$

$$X\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 5 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$$

$$\boxed{X = \begin{bmatrix} -5 & 10 \\ 4 & -6 \end{bmatrix}}$$

Matrix Algebra

$$AX + B = CX$$

$$-AX + AX + B = -AX + CX$$

$$O + B = CX - AX$$

$$B = CX - AX$$

$$B = (C-A)X$$

$$(C-A)^{-1}B = \boxed{\underbrace{(C-A)^{-1}(C-A)}_{I}}X$$

$$(C-A)^{-1}B = X$$

Note on ① Does a Matrix have an inverse?

We need two things..

(a) The matrix <u>must be</u> square

(b) read about <u>determinants</u>.

A is 2x2 $\quad A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$$\det(A) = a_{11}a_{22} - a_{21}a_{12}$$

$$\left[\begin{array}{cc|cc} a_{11} & a_{12} & 1 & 0 \\ a_{21} & a_{22} & 0 & 1 \end{array}\right]$$

if $\det(A) = 0 \rightarrow$ <u>No Inv!</u>

$\det(A) \neq 0 \rightarrow$ <u>have an Inv!</u>

$$\left[\begin{array}{cc|cc} 1 & 0 & b_{11} & b_{12} \\ 0 & 1 & b_{21} & b_{22} \end{array}\right]$$