Math 112

Q's

8.3 # 3

$$A = \begin{bmatrix} 9 & -9 & -5 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ 8 \\ -8 \\ -7 \end{bmatrix}$$

row of A: 1×4; col. of B: 4×1

AB is 1×1

$$AB = 9 \cdot 2 + (-9) \cdot 8 + (-5)(-8) + (4)(-7)$$

$$= 18 - 72 + 40 - 28$$

$$= -72 + 30 = -42$$

$$BA = \begin{bmatrix} 2 \\ 8 \\ -8 \\ -7 \end{bmatrix} \begin{bmatrix} 9 & -9 & -5 & 4 \end{bmatrix} = \begin{bmatrix} 18 & -18 & -10 & 8 \\ 72 & -72 & -40 & 32 \\ -72 & 72 & 40 & -32 \\ -63 & 63 & 35 & -28 \end{bmatrix}$$

4×1 $\quad 1 \times 4$ $\quad 4 \times 4$

ex

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \begin{bmatrix} 1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 0 & 2 \\ 2 & -4 & 0 & 4 \\ 3 & -6 & 0 & 6 \\ 4 & -8 & 0 & 8 \end{bmatrix}$$

ex

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \\ 4 & 8 & 12 & 16 \end{bmatrix} \end{array}$$

6.4 #4

$$\begin{bmatrix} 2 & 8 \\ -8 & -7 \end{bmatrix} - X \begin{bmatrix} 9 & -9 \\ -5 & 4 \end{bmatrix} = I.$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 8 \\ -8 & -7 \end{bmatrix} - X \begin{bmatrix} 9 & -9 \\ -5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(x) $A - XB = C \rightarrow -XB = C - A$

$XB = A - C$

$XBB^{-1} = (A-C)B^{-1}$

$X = (A-C)B^{-1}$

Scratch

$A + X = B$

$-A + A + X = B - A$

$0 + X = B - A$

$X = B - A$

$XC = D$

$XCC^{-1} = DC^{-1}$

$X \cdot I = DC^{-1}$

$X = DC^{-1}$

$$\begin{bmatrix} 2 & 8 \\ -8 & -7 \end{bmatrix} - X \begin{bmatrix} 9 & -9 \\ -5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$-X \begin{bmatrix} 9 & -9 \\ -5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 8 \\ -8 & -7 \end{bmatrix}$$

$$-X \begin{bmatrix} 9 & -9 \\ -5 & 4 \end{bmatrix} = \begin{bmatrix} -1 & -8 \\ 8 & 8 \end{bmatrix} \rightarrow X \begin{bmatrix} 9 & -9 \\ -5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 8 \\ -8 & -8 \end{bmatrix}$$

$$X \begin{bmatrix} 9 & -9 \\ -5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 8 \\ -8 & -8 \end{bmatrix}$$

$$X \underbrace{\begin{bmatrix} 9 & -9 \\ -5 & 4 \end{bmatrix} \begin{bmatrix} 9 & -9 \\ -5 & 4 \end{bmatrix}^{-1}}_{I} = \begin{bmatrix} 1 & 8 \\ -8 & -8 \end{bmatrix} \begin{bmatrix} 9 & -9 \\ -5 & 4 \end{bmatrix}^{-1}$$

$$X = \begin{bmatrix} 1 & 8 \\ -8 & -8 \end{bmatrix} \begin{bmatrix} 9 & -9 \\ -5 & 4 \end{bmatrix}^{-1}$$

$\mathcal{O}$

Find inverse?

$$\left[\begin{array}{cc|cc} 9 & -9 & 1 & 0 \\ -5 & 4 & 0 & 1 \end{array}\right] \xrightarrow{\frac{1}{9}R_1} \left[\begin{array}{cc|cc} 1 & -1 & 1/9 & 0 \\ -5 & 4 & 0 & 1 \end{array}\right]$$

$$5R_1 + R_2 = NR_2 \left[\begin{array}{cc|cc} 1 & -1 & 1/9 & 0 \\ 0 & -1 & 5/9 & 1 \end{array}\right] \xrightarrow{-R_2} \left[\begin{array}{cc|cc} 1 & -1 & 1/9 & 0 \\ 0 & 1 & -5/9 & -1 \end{array}\right]$$

$$R_1 + R_2 = NR_1 \left[\begin{array}{cc|cc} 1 & 0 & -4/9 & -1 \\ 0 & 1 & -5/9 & -1 \end{array}\right]$$

$\uparrow$ inv. of $\begin{bmatrix} 9 & -9 \\ -5 & 4 \end{bmatrix}$

check!

$$\underbrace{\begin{bmatrix} 9 & -9 \\ -5 & 4 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} -4/9 & -1 \\ -5/9 & -1 \end{bmatrix}}_{A^{-1}} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{I} \checkmark$$

$$X = \begin{bmatrix} 1 & 8 \\ -8 & -8 \end{bmatrix} \begin{bmatrix} 9 & -9 \\ -5 & 4 \end{bmatrix}^{-1}$$

and $$\begin{bmatrix} 9 & -9 \\ -5 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} -4/9 & -1 \\ -5/9 & -1 \end{bmatrix}$$

So $$X = \begin{bmatrix} 1 & 8 \\ -8 & -8 \end{bmatrix} \begin{bmatrix} -4/9 & -1 \\ -5/9 & -1 \end{bmatrix} = \boxed{\begin{bmatrix} -44/9 & -9 \\ 8 & 16 \end{bmatrix}}$$

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if $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ A^{-1}?

$$\left[\begin{array}{cc|cc} a & b & 1 & 0 \\ c & d & 0 & 1 \end{array}\right] \overset{\text{a lot of work}}{=} \left[\begin{array}{cc|cc} 1 & 0 & \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ 0 & 1 & \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{array}\right]$$

$$A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$ ✱

ex $A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$ $\quad 1 \cdot 6 - 2 \cdot 3 = 0$

So A^{-1} d.n.e.

Exam on Wed 12 probs @ 10 pts 110 pts = 100%

11.1 $A\sin(\omega x + \phi) + B$

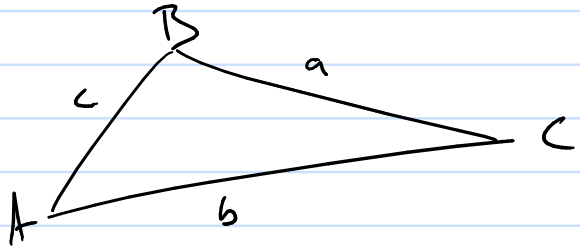
1 prob given information about the graph.

(ex) Amplitude, period, phase, midline.

→ write eqn

11.2 Law of Sines

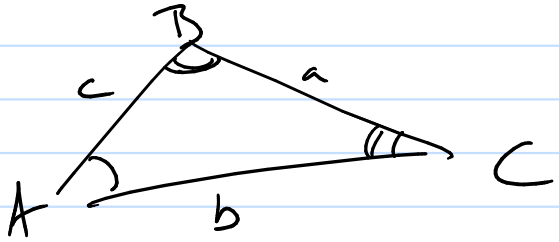
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



2 probs given information → find size/shape of triangle

11.3 Law of Cosines

2 probs



$$a^2 = b^2 + c^2 - 2bc\cos A$$

given info → find size/shape of triangle.

11.4 Polar Coord.

2 probs

(1) given coord. in rect. → find polar

given coord. in polar → find rect.

(2) from (r,θ) to (x,y) of an equation

8.1
8.2

3 probs

① Solve system by substitute

② Solve by elimination

③ Solve by aug. matrix.

ex

$$x + y + z = 3$$
$$x - y + z = 1$$
$$2x - y - z = 0$$

Soln. is $(1,1,1)$

① eq-1 use $x \rightarrow x = 3 - y - z$

eq-2 $(3-y-z) - y + z = 1$
eq-3 $2(3-y-z) - y - z = 0$

$\Rightarrow$

$$-2y = -2$$
$$-3y - 3z = -6$$

③

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & -1 & 1 & 1 \\ 2 & -1 & -1 & 0 \end{array}\right]$$

8.3 (1 prob) Matrix Arithmetic

ex

$$\begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix} + 2\begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$$

8.4 Inverses (1 prob)

① are two given matrices inverses?

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & -1 & 1 & 1 \\ -2 & -1 & -1 & 0 \end{array}\right] \begin{array}{l} -r_1 + r_2 = \text{NR}_2 \\ 2r_1 + r_3 = \text{NR}_3 \end{array} \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -2 & 0 & -2 \\ 0 & -3 & -3 & -6 \end{array}\right] \begin{array}{l} -\frac{1}{2}r_2 = \text{NR}_2 \\ -\frac{1}{3}r_3 = \text{NR}_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 2 \end{array}\right] \quad -r_2 + r_3 = \text{NR}_3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right] \quad \begin{array}{l} x+y+z=3 \\ y=1 \\ z=1 \end{array}$$

So

$$\boxed{\begin{array}{l} x=1 \\ y=1 \\ z=1 \end{array}}$$

(example multiplication)

$$\begin{bmatrix} 1 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} = \boxed{\begin{bmatrix} 1\cdot 1 + (-1)(-1) + (3)(2) \end{bmatrix}}$$

$$= \begin{bmatrix} 1+1+6 \end{bmatrix} = \begin{bmatrix} 8 \end{bmatrix}$$