Math 112

$-\frac{2}{3}\pi$

Q's

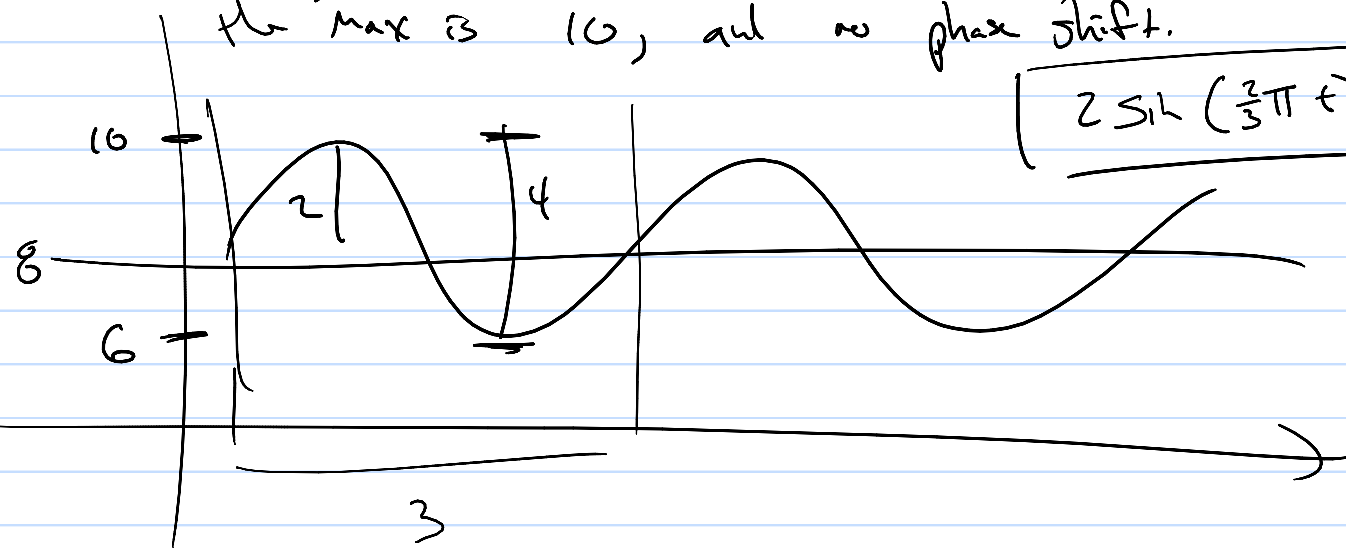
Properties of the Sinusoid $S(t) = A\sin(\omega t + \phi) + B$

2 0 8

- The **amplitude** is $|A|$
- The **angular frequency** is ω and the **ordinary frequency** is $f = \frac{\omega}{2\pi}$
- The **period** is $T = \frac{1}{f} = \frac{2\pi}{\omega}$ $\Rightarrow$ $\frac{2\pi}{\omega} = 3$ so $\omega = \frac{2\pi}{3}$
- The **phase** is ϕ and the **phase shift** is $-\frac{\phi}{\omega}$
- The **vertical shift** or **baseline** is B

(ex) you have sinusoidal motion with a period of 3, distance from max to min of 4 where the max is 10, and no phase shift.

$\boxed{2\sin\left(\frac{2}{3}\pi t\right) + 8}$

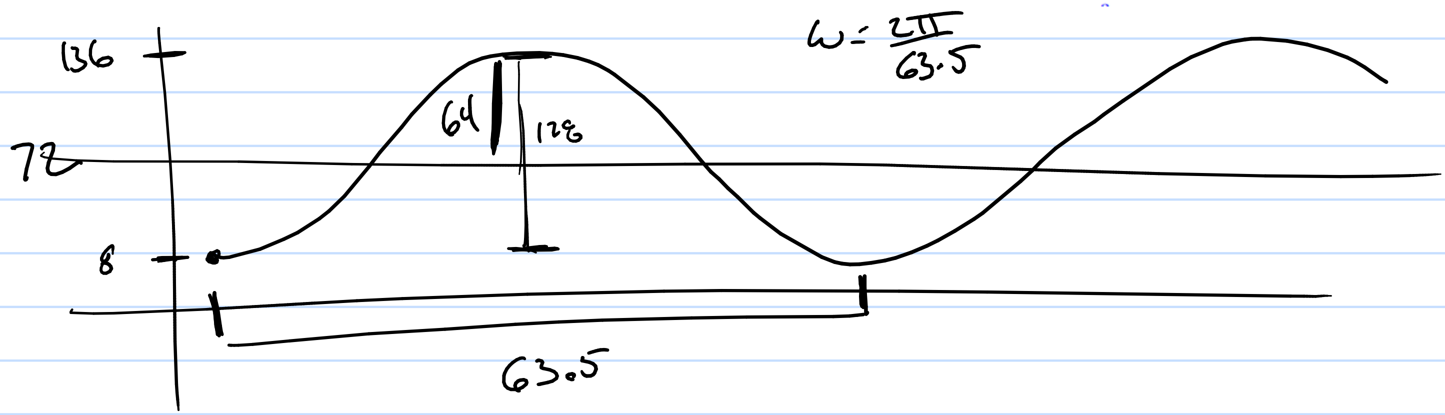


2 min 7 sec = 127 sec

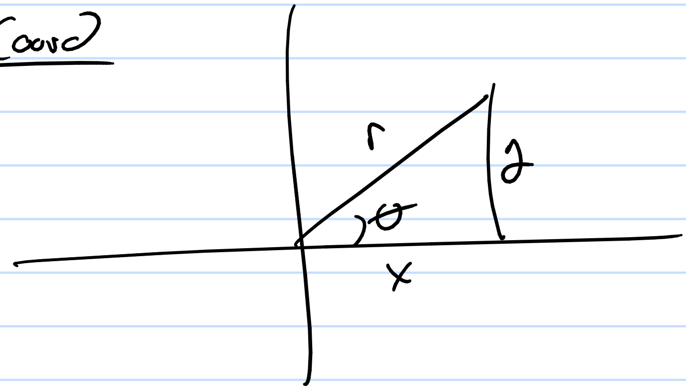
so 1 period = 63.5 sec

Example 11.1.1. Recall from Exercise 55 in Section 10.1 that The Giant Wheel at Cedar Point is a circle with diameter 128 feet which sits on an 8 foot tall platform making its overall height 136 feet. It completes two revolutions in 2 minutes and 7 seconds. Assuming that the riders are at the edge of the circle, ~~find a sinusoid which describes~~ the height of the passengers above the ground t seconds after they pass the point on the wheel closest to the ground.

$\omega = \frac{2\pi}{63.5}$



Polar Coord



$x^2 + y^2 = r^2$

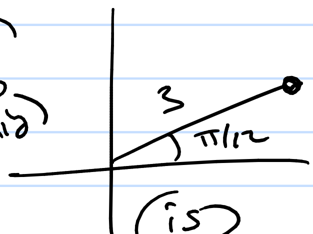
$\tan\theta = y/x$

$x = r\cos\theta$

$y = r\sin\theta$

ex $\quad r = 3 \quad \theta = \pi/12 \qquad x = ? \quad y = ?$

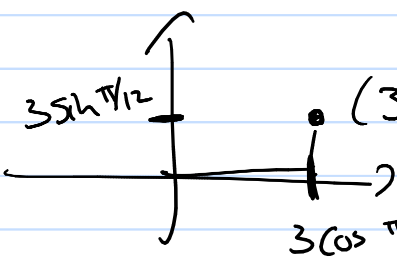
(r, θ) to (x, y)



is

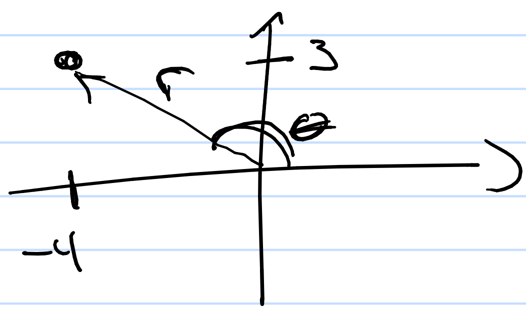
$$\boxed{\begin{aligned} x &= 3\cos\pi/12 \\ y &= 3\sin\pi/12 \end{aligned}}$$

point $(3\cos\pi/12,\ 3\sin\pi/12)$



ex

(x, y) to (r, θ)



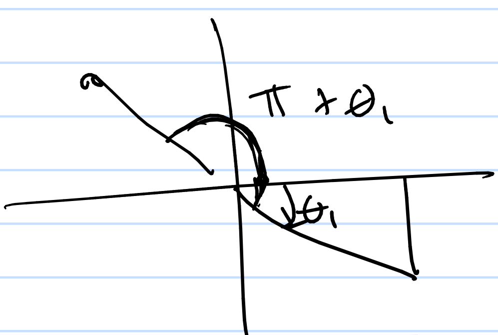
$(-4, 3)$ in (r, θ)

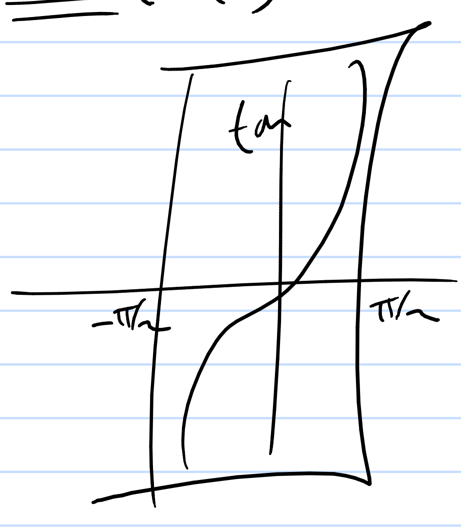
$(-4)^2 + (3)^2 = r^2$

$r^2 = 25$

$r = \pm 5$

$\tan\theta_1 = \frac{3}{-4} \qquad \theta_1 = \underline{\underline{\arctan}}\left(-\frac{3}{4}\right)$





$$\boxed{\left(5,\ \arctan(-3/4) + \pi\right)}$$

ex $\quad r = 3\sin\theta$ in x, y's

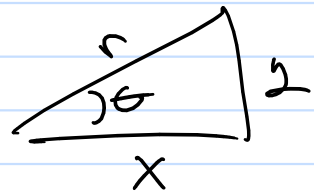
$x = r\cos\theta$

$y = r\sin\theta$

$x^2+y^2=r^2$

$\tan\theta = y/x$

$r\cdot r = 3\left(\frac{y}{\cancel{r}}\right)\cancel{r}$



$r^2 = 3y$

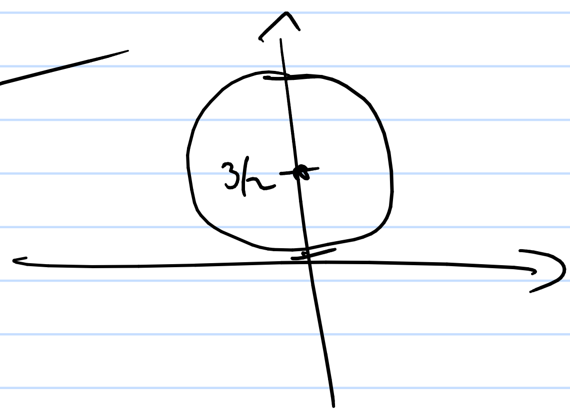
$\boxed{x^2+y^2=3y}$

$\sin\theta = \frac{y}{r}$

$x^2+(y^2-3y)=0$

$x^2+\left(y-3y+\left(\frac{3}{2}\right)^2\right) = \left(\frac{3}{2}\right)^2$

$\boxed{x^2+(y-3/2)^2=\left(\frac{3}{2}\right)^2}$



Complete the square

$x^2+2x+\left(\frac{2}{2}\right)^2 = 2+\left(\frac{2}{2}\right)^2$

$\underbrace{x^2+2x+1} = 3$

$(x+1)^2 = 3$

$(x)^2+2x+(1)^2$

$= (x+1)^2$ &

perfect square trinomials

Solve:

$$\begin{aligned} x + 2y - z &= -2 \\ 2x - y - 2z &= 1 \\ 3x + z &= 4 \end{aligned}$$

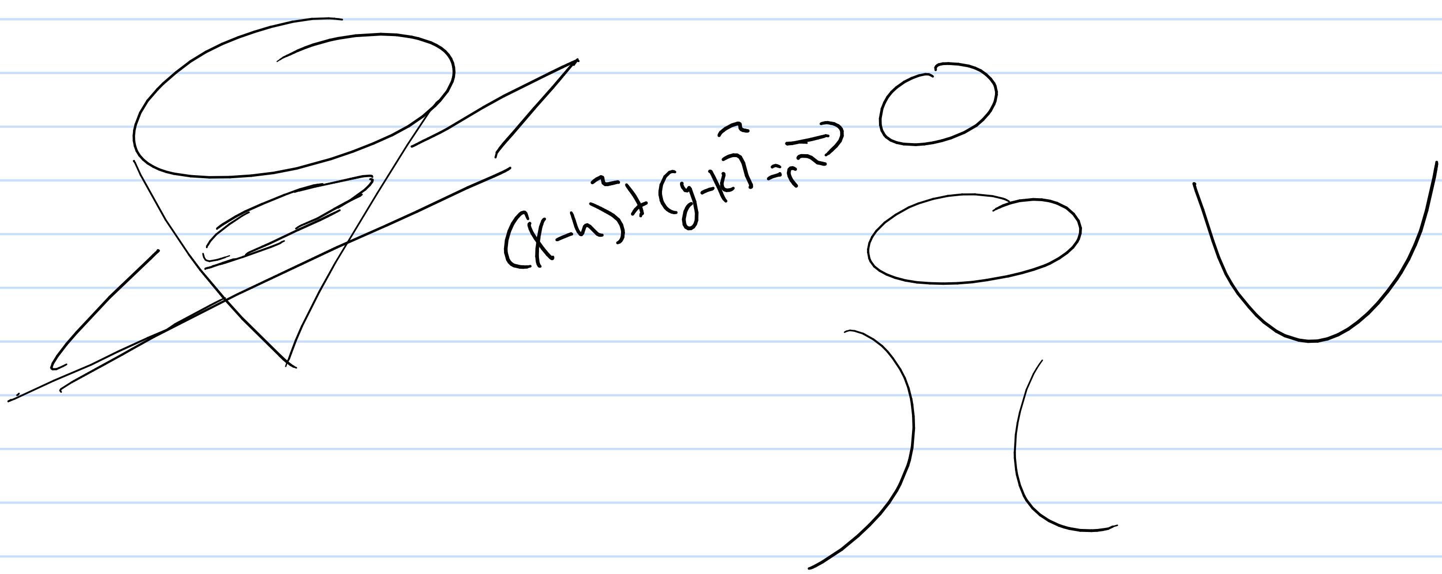
$(1, -1, 1)$ ans.

$$\rightarrow \left[\begin{array}{ccc|c} \boxed{1} & 2 & -1 & -2 \\ 2 & -1 & -2 & 1 \\ 3 & 0 & 1 & 4 \end{array}\right] \begin{array}{l} -2r_1 + r_2 = Nr_2 \\ -3r_1 + r_3 = Nr_3 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & -1 & -2 \\ 0 & \boxed{-5} & 0 & 5 \\ 0 & -6 & 4 & 10 \end{array}\right]$$

$$\begin{array}{l} -\frac{1}{5} r_2 = Nr_2 \\ \frac{1}{2} r_3 = Nr_3 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & -1 & -2 \\ 0 & \boxed{1} & 0 & -1 \\ 0 & -3 & 2 & 5 \end{array}\right] \quad 3r_2 + r_3 = Nr_3 \quad \left[\begin{array}{ccc|c} 1 & 2 & -1 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 2 & 2 \end{array}\right]$$

$$\frac{1}{2} r_3 = Nr_3 \quad \left[\begin{array}{ccc|c} 1 & 2 & -1 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right] \begin{array}{l} \rightarrow x + 2(-1) - 1(1) = -2 \\ \rightarrow y = -1 \\ \rightarrow z = 1 \end{array} \quad x = 1$$

$$(x, y, z) = \boxed{(1, -1, 1)}$$



Sequence / Series

1) chain of elements: 1st, 2nd, 3rd, 4th, ...

How to make them?

① Function: on $n=1, n=2, n=3, n=4, n=5, \ldots$

Sequence is now: $f(1), f(2), f(3), \ldots$

$\boxed{\text{ex}}$ $f(n) = 2n+1$; $n = 1, 2, 3, 4, \ldots$

$f(1) = 3$
$f(2) = 5$
$f(3) = 7$
$f(4) = 9$
$\vdots$

$\boxed{\text{Seq: } 3, 5, 7, 9, \ldots}$

Notation: $\{\text{function}\}_{n=\text{start}}^{\text{end}}$

ex $\{n^2-1\}_{n=2}^{6} = \boxed{\overset{2^2-1}{3}, \overset{3^2-1}{8}, \overset{4^2-1}{15}, \overset{5^2-1}{24}, \overset{6^2-1}{35}}$