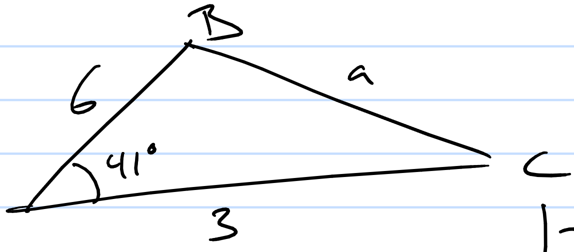
Math 112



$$\boxed{a = \sqrt{3^2 + 6^2 - 2\cdot 3\cdot 6\cos 41^\circ}}$$

$$\boxed{B = \arcsin\left(3\,\frac{\sin 41^\circ}{\sqrt{45 - 36\cos 41^\circ}}\right)}$$

$41^\circ + B + C = 180^\circ$

$$\boxed{C = 139^\circ - \arcsin\left(3\,\frac{\sin 41^\circ}{\sqrt{45 - 36\cos 41^\circ}}\right)}$$

$$\begin{bmatrix} 1 & -1 & 3 \\ 2 & 1 & 4 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \\ 1 & -1 & 2 \\ 3 & 1 & 6 \end{bmatrix} = \begin{bmatrix} \boxed{10} & & \\ & & \\ & & \end{bmatrix}$$

Sequences and Series

ex $\{2n+1\}_{n=2}^{5} = 5, 7, 9, 11$

function that make the seq.

seq.

Notation for a seq.

b/c we can use functions..

[ex] $f(n) = 2n+1$ where $n = 2, 3, 4, 5$

sub-notation: $a_n = 2n+1$ where $n = 2, 3, 4, 5$

$$a_2 = 2\cdot 2+1 = 5$$
$$a_3 = 2\cdot 3+1 = 7$$
$$a_4 = 2\cdot 4+1 = 9$$
$$a_5 = 2\cdot 5+1 = 11$$

in general $\{a_n\}_{n=0}^{\infty}$ has the seq $a_0, a_1, a_2, a_3, \ldots$

---

going from rules to the sequence.

① Functional or closed form.

$\{2^n - 3n\}_{n=0}^{\infty}$ is $2^0 - 3\cdot 0,\ 2^1 - 3\cdot 1,\ 2^2 - 3\cdot 2,\ 2^3 - 3\cdot 3, \ldots$

$1, -1, -2, -1, 4, \ldots$

Ones to know $\{n\}_{n=0}^{\infty}$ is $0, 1, 2, 3, 4, \ldots$

Squares: $\{n^2\}_{n=0}^{\infty}$ is $0, 1, 4, 9, 16, \ldots$

cubes: $\{n^3\}_{n=0}^{\infty}$ is $0, 1, 8, 27, 64, \ldots$

arithmetic: $\{dn+c\}_{n=0}^{\infty}$ is $c, c+d, c+2d, c+3d, \ldots$

(ex) $\{3n+1\}_{n=0}^{\infty}$ is $\underbrace{3\cdot 0+1}_{1},\ \underbrace{3\cdot 1+1}_{4},\ \underbrace{3\cdot 2+1}_{7},\ 3\cdot 3+1$

$1, 4, 7, 10, 13, 16, 19, 22, \ldots$ $\quad +3 \quad +3 \quad +3$

geometric: $\{ar^n\}_{n=0}^{\infty}$ is $ar^0, ar^1, ar^2, ar^3, \ldots$

$$a \xrightarrow{\times r} ar \xrightarrow{\times r} ar^2, ar^3, \ldots$$

---

factorial. $\{n!\}_{n=0}^{\infty}$ is $0!, 1!, 2!, 3!, 4!, 5!, \ldots$

is $1, 1, 2\cdot1, 3\cdot2\cdot1, \underline{4\cdot3\cdot2\cdot1}$

is $1 \xrightarrow{\times 1} 1 \xrightarrow{\times 2} 2 \xrightarrow{\times 3} 6 \xrightarrow{\times 4} 24, \ldots$

---

ex $\{n^2\}_{n=0}^{\infty}$ $\boxed{0} \xrightarrow{+1} 1 \xrightarrow{+3} 4 \xrightarrow{+5} 9 \xrightarrow{+7} \underset{4^2}{16}, \underset{5^2}{25}, 36, \ldots$

---

open forms or recursive or inductive rules

Step 1 give starting value(s) $\boxed{\text{called the Basis step}}$

Step 2 $\boxed{\text{called the recursive or inductive step}}$

need an equation where

New value = expression on old values.

ex $\{2+5n\}_{n=0}^{\infty}$ is $2, 7, 12, 17, 22, 27, \ldots$

closed rule for seq.

$2 \xrightarrow{+5} 7 \xrightarrow{+5} 12$, $17 = 2+5\cdot 3$

recursive.

$a_0 = 2$

$$a_1 = a_0 + 5$$
$$a_2 = a_1 + 5$$
$$a_3 = a_2 + 5$$
$$\vdots$$

$\rightarrow \boxed{a_n = a_{n-1} + 5}$

recursive formula

ex $a_0 = 2$, $a_n = a_{n-1} + 5$ is $2, 7, 12, 17, 22, 27, 32, \ldots$

$2 \xrightarrow{+5} 7 \xrightarrow{+5} 12$

ex $a_0 = 3$

$a_n = 2a_{n-1} + 1$

seq is $3, 7, 15, 31, 63, \ldots$

$3 \xrightarrow{2\cdot 3+1} 7 \xrightarrow{2\cdot 7+1} 15 \xrightarrow{2\cdot 15+1} 31$

compare to $\{2^n\}_{n=0}^{\infty}$ is $1, 2, 4, 8, 16, 32, 64, \ldots$

ex $a_0 = 0$ $a_1 = 1$, $a_n = a_{n-1} + a_{n-2}$ → $0, 1, 1, 2, 3, 5, 8, 13, \ldots$

$0 + 1 = 1$, $1 + 1 = 2$

to do ① rule → seq.

② analyse a seq to guess the rule.

Next: add the numbers of a seq. } **Series**
