Math 112

Exam Makeup   Due Monday

→ call me if you didn't get a blank exam 4 on Friday.

Seq's / Series

(ch 9   9.1, 9.2, 9.4)

seq: $\{a_n\}$ series: $\sum a_n$ binomial thm: $(a+b)^n =$

Seq's $\{a_n\}_{n=1}^{\infty}$ is $a_1, a_2, a_3, a_4, a_5, \ldots$

closed   ex $\{2n+3\}_{n=1}^{\infty}$ is $5, 7, 9, 11, 13, \ldots$ (+2, +2, +2)

arithmetic seq

ex $\{a_n\}_{n=0}^{\infty}$ is

$a_0, a_1, a_2, \ldots$

$1, 4, \boxed{7}, 10, 13, 16, 19, \ldots$

$n=2 \quad a_2 = 7$

arithmetic $\{dn+c\}_{n=0}^{\infty}$ is $c, c+d, c+2d, c+3d, \ldots$ (+d, +d)

1) Common difference $1, 4, 7, 10, 13, 16, 19, \ldots$

subtract

$\boxed{3}\ 3\ 3\ 3$

$4-1 \quad 7-4 \quad 10-7$

$3 =$ common difference

so subtract neighbors → same?

ex $\boxed{1}, 4, 7, 10, 13, 16, 19, 22, \ldots$

test: Subtract neighbors

$4-1=3$
$7-4=3$
$10-7=3$
$\vdots$

common diff. of 3 for an arithmetic seq.

Formula: $\{\underbrace{3n+c}_{a_n}\}_{n=0}^{\infty}$ $\quad a_0, a_1, a_2, \ldots$

$a_0 = 1$

$a_0 = 1$ is $3 \cdot 0 + c = 1$

$n=0$ $\quad$ so $c=1$

$$\boxed{\{3n+1\}_{n=0}^{\infty} \text{ is } 1, 4, 7, \ldots}$$

$1, 4, 7, 10, 13, \ldots$

$\{a_n\}_{n=1}^{\infty}$ is $a_1, a_2, a_3, \ldots$

$\{3n+c\}_{n=1}^{\infty}$ $\quad a_1 = 1, \; a_2 = 4, \; a_3 = 7, \ldots$

$3 \cdot 1 + c = 1$ so $c = -2$

$\{3n-2\}_{n=1}^{\infty}$ is $1, 4, 7, \ldots$

Open form

$1, 4, 7, 10, 13, \ldots$ ($+3$)

$a_1 = 1$

$a_n = a_{n-1} + 3$

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geometric $\{ar^n\}_{n=0}^{\infty}$ $a, ar, ar^2, ar^3, \ldots$

ex $\{2^n\}_{n=0}^{\infty}$ $1, 2, 4, 8, 16, \ldots$

common difference? $1 \quad 2 \quad 4$ not common diff.

common division? $\frac{2}{1} = 2 \quad \frac{4}{2} = 2 \quad \frac{8}{4} = 2$

common product to get next term

$1, 2, 4, \ldots$ ($\times\boxed{2}$, $\times 2$)

$\boxed{2} = r$ of geometric seq.

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ex $-2, \frac{2}{3}, -\frac{2}{9}, \frac{2}{27}, \ldots$

$\dfrac{\frac{2}{3}}{\frac{-2}{1}} = \frac{\cancel{2}}{3} \cdot \frac{1}{-\cancel{2}} = -\frac{1}{3}$ | $\dfrac{-2/9}{2/3} = -\frac{\cancel{2}}{\cancel{9}_3} \cdot \frac{\cancel{3}}{\cancel{2}} = -\frac{1}{3}$

$\dfrac{\frac{2}{27}}{\frac{-2}{9}} = -\frac{1}{3}$

So $r = (-1/3)$ for a geometric seq.

$\{a(-\frac{1}{3})^n\}_{n=0}^{\infty}$ is $-2, 2/3, -2/9, \ldots$ (a_0: 1) (a: 1)

$\left\{a\left(-\frac{1}{3}\right)^n\right\}_{n=0}^{\infty}$ $\quad \underset{a_0}{-2}, \frac{2}{3}, -\frac{2}{9}, \ldots$

$$a\left(-\tfrac{1}{3}\right)^0 = -2$$

$$a = -2$$

$\therefore$ $\boxed{\left\{-2\left(-\frac{1}{3}\right)^n\right\}_{n=0}^{\infty} \text{ is } -2, \frac{2}{3}, -\frac{2}{9}, \ldots}$

therefore

open form

$$\boxed{\begin{aligned} a_1 &= -2 \\ a_n &= \left(-\tfrac{1}{3}\right) \cdot a_{n-1} \end{aligned}}$$

Series : add the values of a sequence.

ex $\quad 0.999999\ldots$

$$= \frac{9}{10} + \frac{9}{100} + \frac{9}{1000} + \frac{9}{10{,}000} + \cdots$$

$$= 9\left(\tfrac{1}{10}\right) + 9\left(\tfrac{1}{10}\right)^2 + 9\left(\tfrac{1}{10}\right)^3 + 9\left(\tfrac{1}{10}\right)^4 + \cdots$$

Notation: $\quad \{a_n\}_{n=1}^{7} \quad a_1, a_2, a_3, \ldots, a_7$ (Seq)

(Sum) $\quad a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 = \sum_{n=1}^{7} a_n$

$$\sum_{n=4}^{11} a_n = a_4 + a_5 + a_6 + a_7 + a_8 + a_9 + a_{10} + a_{11}$$

ex $$\sum_{n=3}^{6} (2n+1) = \underset{n=3}{(7)} + \underset{n=4}{(9)} + \underset{n=5}{(11)} + \underset{n=6}{(13)} = \boxed{40}$$

$$\sum_{k=1}^{n} (1) = \underset{k=1}{(1)} + \underset{k=2}{(1)} + \underset{k=3}{(1)} + \cdots + \underset{k=n}{(1)} = n$$

$$\sum_{k=1}^{n} 3 = 3n$$

$$\sum_{k=1}^{n} k^3$$

$$0.9999\ldots = 1.0$$