Math 112

0.12, 4, 6

$$\frac{0.12+4+6}{3}$$

1 1 = 100% 50% %

6 100 = 0%

1 101

100, 100, ..., 100

0

Q's

1 List the first four terms of each sequence. $(n = 1, 2, 3, \ldots)$

$a_n = 10n - 10$: 0, 10, 20, 30, ...

$b_n = (3)^n$: 3, 9, 27, 81, ...

$c_1 = 5, c_n = 2c_{n-1} + 1$: 5, 11, 23, 47, ...

$$c_n = 2(c_{n-1}) + 1$$

$3 \cdot 27$

$3^4 = (3^2)^2$

$= 9^2 = 81$

$3(27)$

$3(20+7)$

$60 + 21$

81

8 Express the following sum in closed form.

$\sum_{k=1}^{n}(5k-2) =$ [ ]

Note: Your answer should be in terms of n.

9 Compute the sum

$\sum_{i=1}^{n}(2i-1) =$ [ ] .

10 Find the indicated sum.

$$\sum_{n=1}^{6} 3\left(\frac{8}{3}\right)^n =$$

9.2 Sums

upper limit

$$\sum_{k=a}^{n} a_k = a_a + a_{a+1} + \cdots + a_n$$

index   lower limit   seq.

ex

$$\sum_{k=3}^{5} a_k = a_3 + a_4 + a_5$$

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ex $\sum_{k=1}^{3}(5k-2) = (3) + (8) + (13) = \boxed{24}$

how to find sums?

example

① Just Add!

Can't "just add"?

② Use properties and known answers.

Known Answers

(?) $\sum_{k=1}^{n} 1 = \underset{k=1}{(1)} + \underset{k=2}{(1)} + \underset{k=3}{(1)} + \cdots + \underset{k=n}{(1)} = n$

$\sum_{k=1}^{15} 1 = \boxed{15}$

(5) $\boxed{\sum_{k=1}^{n} c = \underbrace{c + c + c + \cdots + c}_{n \text{ c's are here}} = nc}$

(ex) $\sum_{k=1}^{9} 4 = 9 \cdot 4 = \boxed{36}$

(c) $\sum_{k=1}^{n} k = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$

Now if I say this adds to S.

$$S = 1 + 2 + 3 + \cdots + n$$

$$+ \quad S = n + (n-1) + (n-2) + \cdots + 1$$

$$2S = (n+1) + (n+1) + \cdots + (n+1)$$

$$2S = n(n+1)$$

$$S = \frac{n(n+1)}{2}$$

(ex) b/c $\boxed{\sum_{k=1}^{n} k = \frac{n(n+1)}{2}}$

$$\sum_{k=1}^{100} k = \frac{100(101)}{2} = \frac{10{,}100}{2} = \boxed{5{,}050}$$

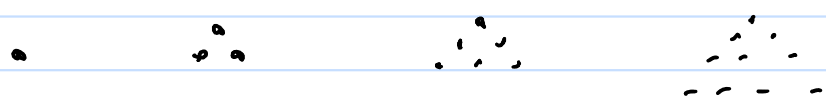
Know:

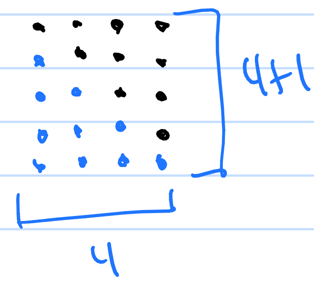
(a) $\displaystyle\sum_{k=1}^{n} 1 = n$

(b) $\displaystyle\sum_{k=1}^{n} c = n \cdot c$

(c) $\displaystyle\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

$1, \quad 1+2, \quad 1+2+3, \quad 1+2+3+4$





Properties if $\displaystyle\sum_{k=n}^{m} a_k$, $\displaystyle\sum_{k=n}^{m} b_k$ both have sums

(a) $\displaystyle\sum_{k=n}^{m} (a_k \pm b_k) = \sum_{k=n}^{m} a_k \pm \sum_{k=n}^{m} b_k$

(b) $\displaystyle\sum_{k=n}^{m} c\, a_k = c \sum_{k=n}^{m} a_k$

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ex $\displaystyle\sum_{k=1}^{n} (5k - 2) = \sum_{k=1}^{n} 5k - \sum_{k=1}^{n} 2 = 5 \sum_{k=1}^{n} k - \sum_{k=1}^{n} 2$

Know

$\displaystyle\sum_{k=1}^{n} 2 = 2n$

$\displaystyle\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

ex $\sum_{k=1}^{n}((5k)-(2)) = \sum_{k=1}^{n}5k - \sum_{k=1}^{n}2 = 5\boxed{\sum_{k=1}^{n}k} - \boxed{\sum_{k=1}^{n}2}$

Know

$$\sum_{k=1}^{n} 2 = 2n \qquad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\boxed{5\left(\frac{n(n+1)}{2}\right) - (2n)} = \boxed{\frac{5}{2}n(n+1) - 2n}$$

$$= \frac{n}{2}\left(5(n+1) - 4\right) = \frac{n}{2}(5n+1)$$

$$\sum_{k=1}^{n}(5k-2) = \boxed{\frac{n(5n-1)}{2}}$$

$$\sum_{k=1}^{1001} k^3 - 2k^2 + k - 4 = \sum_{k=1}^{1001}k^3 - 2\sum_{k=1}^{1001}k^2 + \sum_{k=1}^{1001}k - 4\sum_{k=1}^{1001}1$$

Knowns:

$$\boxed{\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}}$$

$$\boxed{\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2}$$

Geometric

$$\sum_{k=0}^{n} ar^k = a + ar + ar^2 + \cdots + ar^n = ?$$

$$= a\left(\frac{1-r^{n+1}}{1-r}\right) \quad r \neq 1$$

$$S = a + \boxed{ar} + \boxed{ar^2} + \cdots + \boxed{ar^n}$$

$$-(rS = \boxed{ar} + \boxed{ar^2} + ar^3 + \cdots + \boxed{ar^n} + ar^{n+1})$$

$$S - rS = a - ar^{n+1}$$

$$S(1-r) = a(1-r^{n+1})$$

$$S = a\left(\frac{1-r^{n+1}}{1-r}\right) \quad \underline{r \neq 1}$$

$$\boxed{\sum_{k=0}^{n} ar^k = a\left(\frac{1-r^{n+1}}{1-r}\right) \quad (r \neq 1)}$$

8 Express the following sum in closed form.

$\sum_{k=1}^{n} (5k-2) =$ ____

Note: Your answer should be in terms of n.

9 Compute the sum

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$$= \sum_{n=0}^{5} 3\left(\frac{8}{3}\right)^{(n+1)}$$

$$= \sum_{n=0}^{5} 3\left(\frac{8}{3}\right)\left(\frac{8}{3}\right)^n$$

$$= \boxed{\sum_{n=0}^{5} 8\left(\frac{8}{3}\right)^n}$$

$$= \boxed{8\left(\frac{1-\left(\frac{8}{3}\right)^6}{1-8/3}\right)}$$