Math 112

Q's 9.2 #14

14. If you make quarterly payments of **$599.00** into an ordinary annuity earning an annual interest rate of **3.15**%, how much will be in the account after **4** years?

(**Note:** Your answer is a dollar amount and should include a dollar sign)

Know: $\sum_{k=1}^{n} c = cn$, $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$, $\sum_{k=1}^{n} k^2 = \frac{(2n+1)(n)(n+1)}{6}$

$\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$, $\sum_{k=0}^{n} ar^k = a\left(\frac{1-r^{n+1}}{1-r}\right)$

with $\sum_{k=n}^{m}(a_k \pm b_k) = \sum_{k=n} a_k \pm \sum_{k=n} b_k$

$\sum_{k=n}^{m} ca_k = c\sum_{k=n}^{m} a_k$

$$\boxed{\sum_{k=n}^{p} a_k + \sum_{k=p}^{m} a_k = \sum_{k=n}^{m} a_k}$$

$\boxed{i = \frac{r}{n}}$ rate per compounding

Word Problem : Annuity

annual interest rate $= r$
compoundings per year $= n$

new amount = old amount · (with) interest + P

during time of compounding

deposit at end of compounding period

$$A_n = A_{n-1}\left(1 + \text{compounding time interest}\right) + P$$

$$A_n = A_{n-1}(1+i) + P$$

$$A_0 = P$$

$$A_1 = \boxed{P(1+i)^1 + P}$$

$$A_2 = A_1(1+i) + P = \boxed{P(1+i) + P}(1+i) + P$$

$$\rightarrow A_2 = P(1+i)^2 + P(1+i) + P$$

$$A_k = P(1+i)^k + P(1+i)^{k-1} + \cdots + P(1+i)^2 + P(1+i) + P$$

remember:

$$\boxed{\sum_{k=0}^{n} ar^k = \boxed{a + ar + ar^2 + ar^3 + \cdots + ar^n} = a\left(\frac{1-r^{n+1}}{1-r}\right) = a\left(\frac{r^{n+1}-1}{r-1}\right)}$$

Annuity

$$A_k = \boxed{P + P(1+i) + P(1+i)^2 + \cdots + P(1+i)^k}$$

$$= P\left(\frac{1-(1+i)^{k+1}}{1-(1+i)}\right)$$

compoundings

$$= P\left(\frac{(1+i)^{k+1}-1}{(1+i)-1}\right) = P\left(\frac{(1+i)^{k+1}-1}{i}\right)$$

but compoundings $= nt$ w/ $n = \frac{\text{compoundings}}{\text{yr}}$ $t = \text{yr}$

$$A = P\left[\frac{(1+i)^{nt}-1}{i}\right]$$

$n \equiv$ compoundings / yr

$t \equiv$ years

$i = r/n$

$r =$ annual interest rate

$n = 4$ $P = 599$

14 If you make quarterly payments of $599.00 into an ordinary annuity earning an annual interest rate of 3.15%, how much will be in the account after 4 years?

$r = 0.0315$ $t = 4$ $i = \frac{0.0315}{4}$

(**Note:** Your answer is a dollar amount and should include a dollar sign)

$$A = 599\left(\frac{\left(1+\frac{0.0315}{4}\right)^{4\cdot 4}-1}{\frac{0.0315}{4}}\right) \approx \$\,?$$

$d_1 \quad d_2 \quad d_3 \quad d_4 \quad d_5 \quad d_6 \quad d_7 \ldots d_{28}$ (Feb)

(a) $1,000,000

(b) 0.01 0.02 0.04 0.08 0.16 0.32 0.64 ...

on d_7 Total = 1¢ + 2¢ + 4¢ + 8¢ + 16¢ + 32¢ + 64¢

$d_k = 2^{k-1}$

$d_{28} = 2^{27}$¢ $= \$1,342,177.28$

total: 1¢ + 2¢ + 4¢ + ... + 2^{27}¢ $= 2^{28}-1$

9.1

10 For each sequence, find a closed formula for the general term, a_n.

1. $0, 3, 8, 15, 24, \ldots, a_n =$ $\boxed{n^2-1}$, $n=1,2,3,\ldots$

2. $-2, -8, -18, -32, -50, \ldots, a_n =$ $-2n^2$, $n=1,2,3,\ldots$

3. $100, 81, 62, 43, 24, \ldots, a_n =$ ______ .

$\{k^2\} = 1, 4, 9, 16, 25, \ldots$

diff $3 \quad 5 \quad 7 \quad 9$

ratio $4 \quad \frac{9}{4} \quad \frac{16}{9} \quad \frac{25}{16}$?

$0, 3, 8, 15, 24, \ldots$

diff $3 \quad 5 \quad 7 \quad 9$

$-2, -8, -18, -32, -50, \ldots$

diff $-6 \quad -10 \quad -14 \quad -18$

ratio $\frac{-8}{-2} = 4 \quad \frac{-18}{-8} = 9/4 \quad \frac{-32}{-18} = 16/9$

k $\quad 1, 2, 3, 4, 5, \ldots$

k^2 $\quad 1, 4, 9, 16, 25, \ldots$ $\qquad 2k^2$ $\quad 2, 8, 18, 32, 50, \ldots$

k^3 $\quad 1, 8, 27, 64, 125, \ldots$ $\qquad -2k^2$ $\quad -2, -8, -18, -32, -50, \ldots$

$k!$ $\quad 1, 2, 6, 24, 120, \ldots$

$100, 81, 62, 43, 24, \ldots$

$81-100 = -19 \quad 62-81 = -19 \quad 43-62 = -19$

arithmetic by -19

$-19n + c$

@ $n=1$ it is 100

$100 = -19(1) + c$

$c = 119$

$a_n = \boxed{-19n+119}$ $\quad n=1,2,3,\ldots$

~~$n=0,1,2,\ldots$ $a_n = -19n + 100$~~

9.4 Binomial th^m

Factorial:

$$\boxed{\begin{aligned} 0! &= 1 \\ n! &= n\,(n-1)! \end{aligned}}$$

$0! = 1$

$1! = 1$

$2! = 2 \cdot 1 = 2$

$3! = 3 \cdot 2 \cdot 1 = 6$

$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

$\vdots$

$10! = \boxed{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

$(a+b)^n$

$(a+b)(a+b)(a+b) \cdots (a+b)$

$(3x-y)^{17} =$ ______