

Math 112

Q's

1.2 #14

- 14 If you make quarterly payments of \$599.00 into an ordinary annuity earning an annual interest rate of 3.15%, how much will be in the account after 4 years?

(Note: Your answer is a dollar amount and should include a dollar sign)

Know:

$$\sum_{k=1}^n c = cn, \quad \sum_{k=1}^n k = \frac{n(n+1)}{2}, \quad \sum_{k=1}^n k^2 = \frac{(2n+1)(n)(n+1)}{6}$$

$$\sum_{k=1}^n k^3 = \left(\frac{n(n+1)}{2}\right)^2, \quad \sum_{k=0}^n ar^k = a \left(\frac{1-r^{n+1}}{1-r}\right)$$

rule

$$\sum_{k=n}^m (a_k \pm b_k) = \sum_{k=n}^m a_k \pm \sum_{k=n}^m b_k$$

$$\sum_{k=n}^m ca_k = c \sum_{k=n}^m a_k$$

$$\sum_{k=n}^p a_k + \sum_{k=p}^m a_k = \sum_{k=n}^m a_k$$

$$i = \frac{r}{n}$$

rate per compounding

Word Problem : Annuity

annual interest rate = r
 compounding per year = n

new amount = old amount \cdot interest + P

with \downarrow

duration of compounding \uparrow

deposit at end of compounding period \uparrow

$$A_n = A_{n-1} \left(1 + \frac{\text{compounding rate}}{n}\right) + P$$

$$A_n = A_{n-1}(1+i) + P$$

$$A_0 = P$$

$$A_1 = P(1+i) + P$$

$$A_2 = A_1(1+i) + P = [P(1+i) + P](1+i) + P$$

$$\rightarrow A_2 = P(1+i)^2 + P(1+i) + P$$

$$A_k = P(1+i)^k + P(1+i)^{k-1} + \dots + P(1+i)^2 + P(1+i) + P$$

remember:

$$\sum_{k=0}^n ar^k = a + ar + ar^2 + ar^3 + \dots + ar^n$$

$$= a \left(\frac{1 - r^{n+1}}{1 - r} \right) = a \left(\frac{r^{n+1} - 1}{r - 1} \right)$$

by analogy

$$A_k = P + P(1+i) + P(1+i)^2 + \dots + P(1+i)^k$$

$$= P \left(\frac{1 - (1+i)^{k+1}}{1 - (1+i)} \right)$$

$$= P \left(\frac{(1+i)^{k+1} - 1}{(1+i) - 1} \right) = P \left(\frac{(1+i)^{k+1} - 1}{i} \right)$$

compounding \rightarrow

\downarrow

but compounding = nr and n = $\frac{\text{compounding}}{\text{yr}}$ t = yr

$$A = P \left[\frac{(1+i)^{nt} - 1}{i} \right]$$

$n \equiv$ compounds / yr

$t \equiv$ years

$i =$ %

$r =$ annual interest rate

$n = 4$

$P = 599$

14

If you make quarterly payments of \$599.00 into an ordinary annuity earning an annual interest rate of 3.15%, how much will be in the account after 4 years?

$r = 0.0315$

$t = 4$

$i = \frac{0.0315}{4}$

(Note: Your answer is a dollar amount and should include a dollar sign)

$$A = 599 \left(\frac{(1 + \frac{0.0315}{4})^{4 \cdot 4} - 1}{\frac{0.0315}{4}} \right) \approx \$?$$

$d_1 \quad d_2 \quad d_3 \quad d_4 \quad d_5 \quad d_6 \quad d_7 \quad \dots \quad d_{28}$

(Feb)

(a)

\$1,000,000

(b) 0.01 0.02 0.04 0.08 0.16 0.32 0.64 ...

on d_7

total = $1d + 2d + 4d + 8d + 16d + 32d + 64d$

$d_k = 2^{k-1}$

$d_{28} = 2^{27} d = \$1,342,177.28$

total: $1d + 2d + 4d + \dots + 2^{27} d = 2^{28} - 1$

9.1

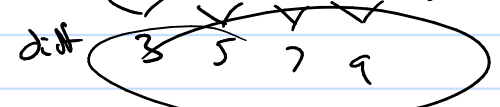
10 For each sequence, find a closed formula for the general term, a_n .

1. $0, 3, 8, 15, 24, \dots, a_n = n^2 - 1, n=1, 2, 3, \dots$

2. $-2, -8, -18, -32, -50, \dots, a_n = -2n^2, n=1, 2, 3, \dots$

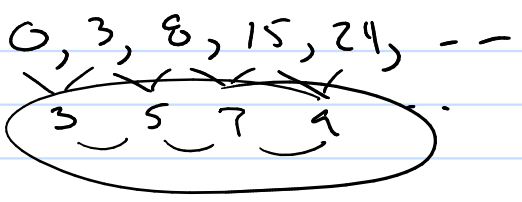
3. $100, 81, 62, 43, 24, \dots, a_n =$

$\{k^2\} = 1, 4, 9, 16, 25, \dots$

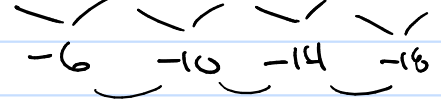


ratio $4, \frac{6}{4}, \frac{8}{6}, \frac{10}{8}, ?$

diff



$-2, -8, -18, -32, -50, \dots$



ratio $\frac{-8}{-2}, \frac{-18}{-4}, \frac{-32}{-6}$
 $4, \frac{9}{4}, \frac{16}{6}$

k $1, 2, 3, 4, 5, \dots$

k^2 $1, 4, 9, 16, 25, \dots$

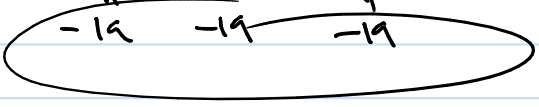
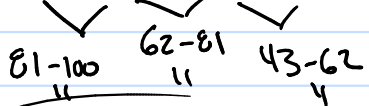
k^3 $1, 8, 27, 64, 125, \dots$

$k!$ $1, 2, 6, 24, 120, \dots$

$2k^2$ $2, 8, 18, 32, 50, \dots$

$-2k^2$ $-2, -8, -18, -32, -50, \dots$

$100, 81, 62, 43, 24, \dots$



$-kn + c$

@ $n=1$ it is 100

$100 = -19(1) + c$

$c = 119$

↑ arithmetic by -19

$a_n = -19n + 119$

$n=1, 2, 3, \dots$



9.4 Binomial th^n

Factorial :

$$\begin{aligned} 0! &= 1 \\ n! &= n(n-1)! \end{aligned}$$

$$0! = 1$$

$$1! = 1$$

$$2! = 2 \cdot 1 = 2$$

$$3! = 3 \cdot 2 \cdot 1 = 6$$

$$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$

\vdots

$$10! = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

$$(a+b)^n$$

$$(a+b)(a+b)(a+b) \dots (a+b)$$

$$(3x-y)^{17} = \underline{\hspace{15em}}$$