Math 112

Q's Write the first four terms of the binomial expansion of $(a + 4b)^{12}$.

<u>Binomial thm</u>

$$(\square + \Delta)^n = \underbrace{(\square + \Delta)(\square + \Delta)\cdots(\square + \Delta)}_{n\text{-factors}}$$

$$= ?$$

(1) <u>Factorial</u> : !

$0! = 1$ $\quad$ $2! = 2\cdot 1$ $\quad$ $4! = 4\cdot 3\cdot 2\cdot 2$

$1! = 1$ $\quad$ $3! = 3\cdot 2\cdot 1$

↳ counting this $n!$ ways to arrange n things.

(2) from n objects place them into <u>two</u> groups. r in one group
$n-r$ in second group

$$\frac{n!}{r!\,(n-r)!}$$

$$\underbrace{e_1, e_2, \ldots}_{r} \Big| \underbrace{\ldots, e_n}_{n-r}$$

from n choose r $= \dfrac{n!}{r!\,(n-r)!}$

$$\boxed{C(n,r)} = \frac{n!}{r!\,(n-r)!} \quad \text{or} \quad \boxed{\binom{n}{r}} = \frac{n!}{r!\,(n-r)!}$$

ex from 10 kids choose 3

$C(10,3)$ or $\dbinom{10}{3}$ is $\dfrac{10!}{3!\ \ 7!}$

$$\frac{10!}{3!\,7!} = \frac{10\cdot 9\cdot 8\cdot \cancel{7!}}{3!\,\cancel{7!}} = \frac{10\cdot \cancel{9}^{3}\cdot \cancel{8}^{4}}{\cancel{3}\cdot\cancel{2}} = \boxed{120}$$

ex

$$\binom{11}{2} = \frac{\overset{11\cdot 10}{\cancel{11!}}}{2!\,\cancel{9!}} = \frac{11\cdot\cancel{10}^{5}}{\cancel{2}} = \boxed{55}$$

$$= \frac{11\cdot\cancel{10}^{5}\cdot\cancel{9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}}{(\cancel{2\cdot 1})(\cancel{9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1})}$$

Note: choose: $C(n,r) = \dfrac{n!}{r!\,(n-r)!}$

permutation or pick function

from n pick r with order

$$P(n,r) = \frac{n!}{(n-r)!}$$

ex pick 5 player from the 15 kids.

$$P(15,5) = \frac{15!}{10!}$$

$$(x+y)^3 = (x+y)(x+y)(x+y)$$

$$= (x\cdot x + x\cdot y + y\cdot x + y\cdot y)(x+y)$$

$$= (x^2 + 2xy + y^2)(x+y)$$

$$= x^3 + 2x^2y + xy^2 + x^2y + 2xy^2 + y^3$$

$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

xxx   xxy   xyy   yyy

$$\frac{3!}{3!\,0!} \qquad \frac{3!}{2!\,1!} \qquad \frac{3!}{1!\,2!} \qquad \frac{3!}{0!\,3!}$$

$$= \qquad = \qquad = \qquad =$$

$$1 \qquad 3 \qquad 3 \qquad 1$$

$$(\square + \Delta)^n = \frac{n!}{n!\,0!}\square^n\Delta^0 + \frac{n!}{(n-1)!\,1!}\square^{n-1}\Delta^1 + \frac{n!}{(n-2)!\,2!}\square^{n-2}\Delta^2 + \cdots + \square^0\Delta^n$$

$$\binom{n}{0} \qquad \binom{n}{1} \qquad \binom{n}{2}$$

$$= \sum_{k=0}^{n}\binom{n}{k}\square^{n-k}\Delta^k \qquad (\square+\Delta)^n$$

Q Write the first four terms of the binomial expansion of $(a - 4b)^{12}$.

$$(a)^{12} + \frac{12!}{11!\,1!}(a)^{11}(-4b)^1 + \frac{12!}{10!\,2!}(a)^{10}(-4b)^2 + \frac{12!}{9!\,3!}(a)^9(-4b)^3 + \cdots$$

$$(a)^{12} + \boxed{\frac{12!}{11!\,1!}}(a)^{11}(-4b)^1 + \boxed{\frac{12!}{10!\,2!}}(a)^{10}(-4b)^2 + \frac{12!}{9!\,3!}(a)^9(-4b)^3 + \cdots$$

66

$$= a^{12} - 48\,a^{11}b + 1056\,a^{10}b^2 + \left(-\ ?\ \right) + \cdots$$

$$(x+2y)^4 = 1\cdot x^4 + 4\,x^3(2y) + \frac{4!}{2!\,2!}x^2(2y)^2 + 4\cdot x(2y)^3 + 1(2y)^4$$

$$24 \cdot 3 = 6$$

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for $(\square + \Delta)^n$

Pascal's Triangle

$$\begin{array}{c}
1 \\
1 \quad 1 \\
1 \quad 2 \quad 1 \\
1 \quad 3 \quad 3 \quad 1 \\
1 \quad 4 \quad 6 \quad 4 \quad 1 \\
1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1 \\
1 \quad 6 \quad 15 \quad 20 \quad 15 \quad 6 \quad 1 \\
1 \quad 7 \quad 21 \quad 35 \quad 35 \quad 21 \quad 7 \quad 1
\end{array}$$

$(\square+\Delta)^4$ coef $\rightarrow$ 1 4 6 4 1

$$\binom{4}{0} \quad \binom{4}{1} \quad \binom{4}{2} \quad \binom{4}{3} \quad \binom{4}{4}$$
$$= \quad = \quad = \quad = \quad =$$
$$1 \quad 4 \quad 6 \quad 4 \quad 1$$

Fact:

$$\binom{n-1}{k-1} + \binom{n-1}{k} = \binom{n}{k}$$

ex $(s+t)^7 = \boxed{s^7 + 7s^6t + 21s^5t^2 + 35s^4t^3 + 35s^3t^4 + 21s^2t^5 + 7st^6 + t^7}$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

$\boxed{1 \quad 7 \quad 21 \quad 35 \quad 35 \quad 21 \quad 7 \quad 1}$

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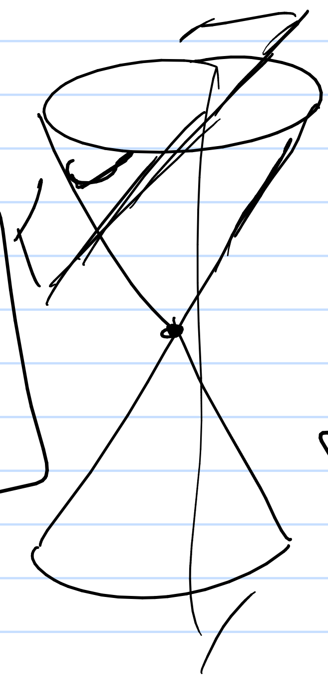
ch 7

7.1

double napped cone



- circles
- parabola
- ellipse
- hyperbola

all from a plane slicing this