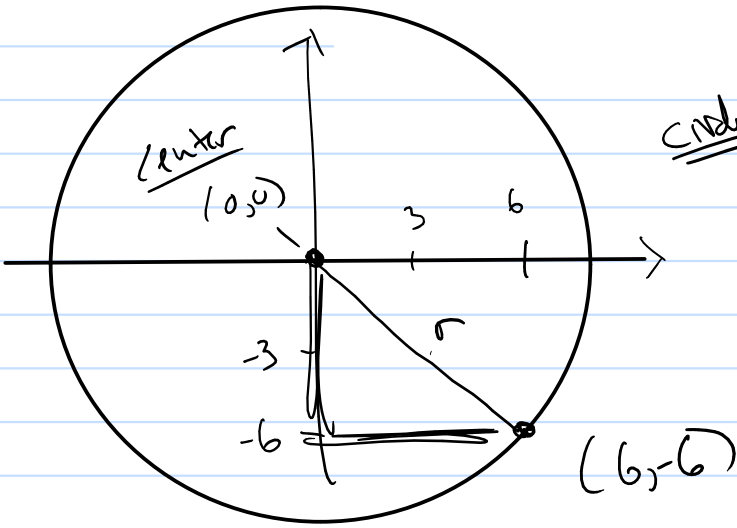
Math 112

Q's

#4

Find an equation of the circle with center at the origin and passing through $(6, -6)$ in the form of

$$(x - \overset{0}{\underset{h}{A}})^2 + (y - \overset{0}{\underset{k}{B}})^2 = \overset{72}{\underset{r^2}{C}}$$



circle $(x-h)^2 + (y-k)^2 = r^2$

center (h,k) ✓

radius r ✓

$6^2 + (-6)^2 = r^2$

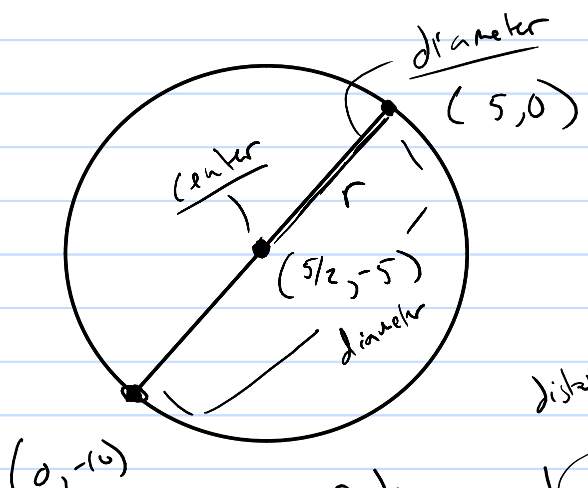
$36 + 36 = r^2$

$r^2 = 72$

$r = 6\sqrt{2}$

#5

Find the standard form for the equation of a circle $(x - \underset{5/2}{h})^2 + (y - \underset{-5}{k})^2 = \underset{\frac{125}{4}}{r^2}$ with a diameter that has endpoints of $(0, -10)$ and $(5, 0)$.



center is midpoint of the endpoints of a diameter

center is $\left(\frac{0+5}{2}, \frac{-10+0}{2}\right)$

$= \left(5/2, -5\right)$

distance from (5,0) to (0,-10)

radius $= \frac{1}{2}$ diameter length

radius is $\frac{1}{2}\sqrt{(5)^2 + (10)^2} = \frac{1}{2}\sqrt{125} = r$

#6

Find the center and radius of the circle whose equation is $\boxed{4x^2 + 10x + 4y^2 - 10y - 1 = 0.}$

$\div 4$

$$x^2 + \frac{5}{2}x \qquad + y^2 - \frac{5}{2}y \qquad - \frac{1}{4} = 0$$

$$x^2 + \frac{5}{2}x + \left(\frac{5}{4}\right)^2 + y^2 - \frac{5}{2}y + \left(\frac{-5}{4}\right)^2 = \frac{1}{4} + \left(\frac{5}{4}\right)^2 + \left(\frac{-5}{4}\right)^2$$

$$\left(x + \frac{5}{4}\right)^2 + \left(y - \frac{5}{4}\right)^2 = \frac{1}{4} + \frac{25}{16} + \frac{25}{16}$$

$\frac{50}{16}$

$\frac{25}{8}$

Center $\left(-\frac{5}{4}, \frac{5}{4}\right)$

$$r = \sqrt{\frac{27}{8}} = \frac{3\sqrt{3}}{2\sqrt{2}} = \frac{3}{4}\sqrt{6}$$

$$\left(x + \frac{5}{4}\right)^2 + \left(y - \frac{5}{4}\right)^2 = \frac{1}{4} + \frac{25}{8} = \frac{27}{8}$$

Making perfect square trinomials

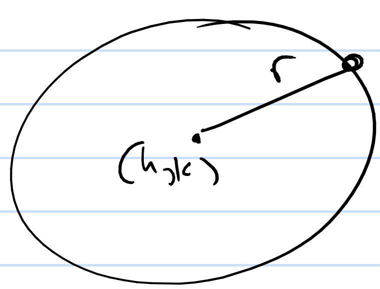
$$(a+b)^2 = (a)^2 + 2ab + (b)^2$$

$$(x+b)^2 = (x)^2 + (2b)x + (b)^2$$

$$\left(x + \frac{c}{2}\right)^2 = x^2 + cx + \left(\frac{c}{2}\right)^2$$

$$x^2 + c \cdot x + \left(\frac{c}{2}\right)^2 = \left(x + \frac{c}{2}\right)^2$$

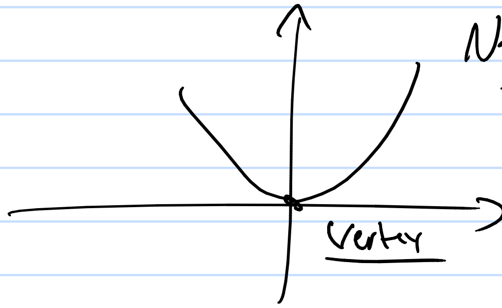
$$(x-h)^2 + (y-k)^2 = r^2$$



Parabola

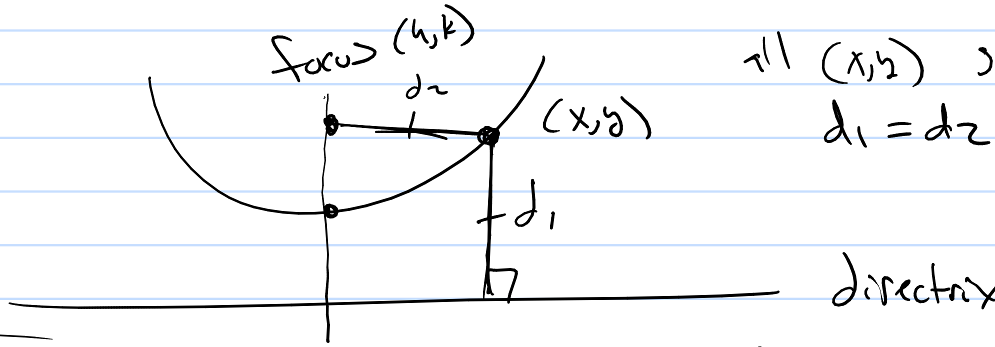
$y = x^2$

before

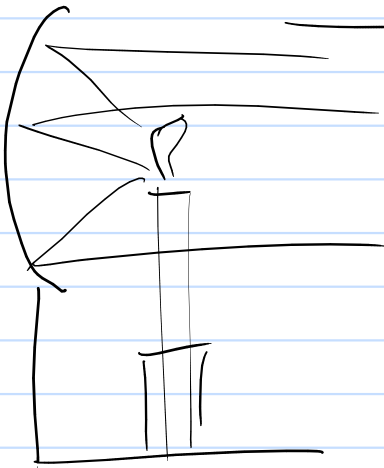


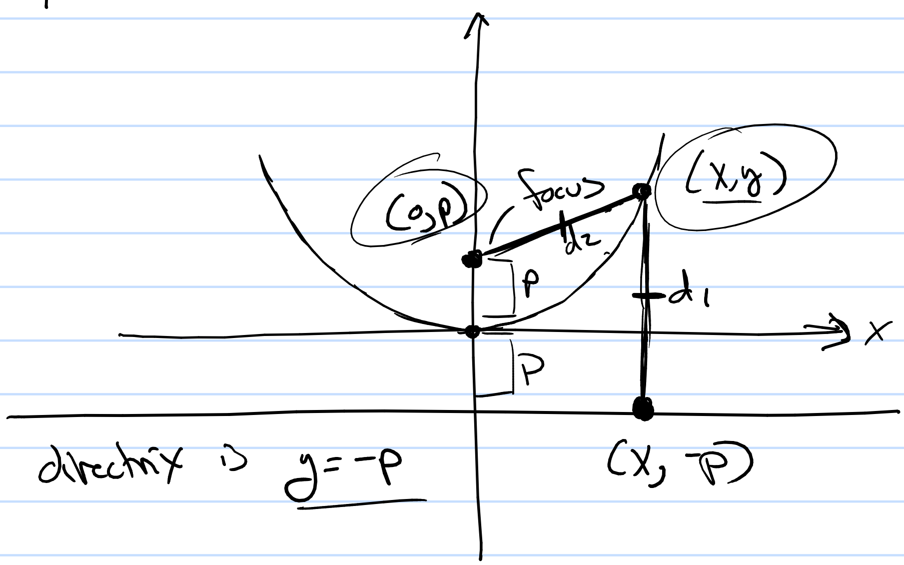
Named this parabola.

Parabola



All (x,y) such that
$d_1 = d_2$





$$d_2^2 = x^2 + (y-p)^2 \qquad d_1^2 = 0^2 + (y+p)^2$$

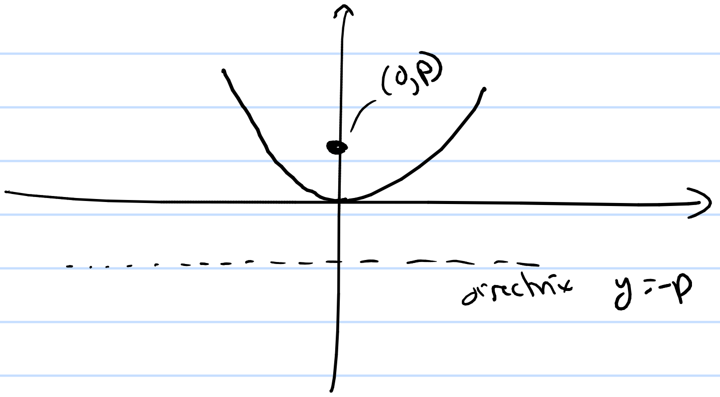
if $d_1 = d_2$ (parabola)

then $d_1^2 = d_2^2$

$$x^2 + (y-p)^2 = (y+p)^2$$

$$x^2 + \cancel{y^2} - 2yp + \cancel{p^2} = \cancel{y^2} + 2yp + \cancel{p^2}$$

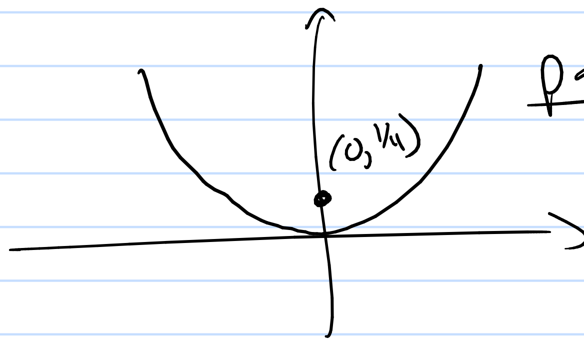
$$x^2 = 4yp \rightsquigarrow y = \frac{1}{4p} x^2$$



eqn $\boxed{y = \frac{1}{4p}x^2}$

Standard form $\boxed{4py = x^2}$

So $\boxed{\text{before}}$ $y = x^2$



parabola.

b/c $y = \left(\frac{1}{4p}\right)x^2$

$$\frac{1}{4p} = 1$$

$$\frac{1}{4} = p$$

$\boxed{Q}$ $1 - 4 + 9 - 16 + 25 - 36 + 49$

Seq: $1, 4, 9, 16, 25, 36, 49$

$1^2, 2^2, 3^2, 4^2, 5^2, 6^2, 7^2$ seq is k^2, $k = 1, 2, \ldots, 7$

Sign?: $+, -, +, -, +, -, +, \ldots$

use $(-1)^{\text{power}}$

b/c $(-1)^1 = -1$

$(-1)^2 = 1$

$(-1)^3 = -1$

$(-1)^4 = 1$

So $+1^2, -2^2, +3^2, -4^2, +5^2, -6^2, +7^2$

$\boxed{(-1)^{k+1}k^2 \;;\; k = 1, 2, 3, 4, 5, 6, 7}$

$$\boxed{\sum_{k=1}^{7} (-1)^{k+1} k^2}$$