

Math 112

Final

Next Wed @ 9am (Dec 8th)

We have had 5 exams.

$$\text{Final} = \text{Exam 1} + \text{Exam 2} + \text{Exam 3} + \text{Exam 4} + \text{Exam 5}$$

(pick 4) (pick 4) (pick 4) (pick 4) (pick 4)

20 probs to get done in 1hr 50min

Below

⊗ means not on final

○ means on final

? means study for final

EXAM 1

- 1a) Plot the points: $(-2,4)$, $(3,1)$, $(-1,-2)$, $(2,-3)$
 1b) Create a table of values and plot the graph of $y = 3x - 1$
 1c) Create a table of values and plot the graph of $y = |x| - 2$
 1d) Create a table of values and plot the graph of $y = x^2 + x - 6$

- 2) For $f(x) = x^2 - \frac{1}{x+1}$ and $g(x) = 1 - x^2$
- What is the domain of $f(x)$? What is the domain of $g(x)$?
 - Find $f(1)$ and $g(2)$
 - Find $f(-2) + g(2)$
 - Find $f(x+h)$. Do not simplify your answer.
 - Find $(f+g)(x)$. Write your answer as a rational function and state the domain.

- 3a) Create individual plots for each of the functions $f(x) = x$, $f(x) = x^2$, $f(x) = x^3$, $f(x) = |x|$, and $f(x) = \frac{1}{x}$.
 3b) Write the function that is $f(x) = \frac{1}{x}$ translated one unit to the left and three units up.
 3c) Write the function that is $f(x) = x^3$ translated two units to the right and one unit down.

Handwritten notes for question 3:

$$y = x \quad y = x^3 \quad y = \frac{1}{x} \quad y = \cos x \quad y = e^x \quad y = \sec x$$

$$y = x^2 \quad y = |x| \quad y = \sin x \quad y = \tan x \quad y = \ln x \quad x^2 + y^2 = r^2$$

- 4) For each function determine if it is symmetric about the y-axis, origin, or neither.

- $f(x) = \frac{1}{x^2} + 2$
- $f(x) = x - x^4$
- $f(x) = x^3 - x$

$$f(-x) = -f(x)$$

vs

$$f(-x) = f(x)$$

- 5a) Find the equation for a line passing through the point $(-2, 1)$ and is perpendicular to the line $y = 3x + 1$. Write your answer in point-slope form.

- 5b) Find the equation for a line passing through the points $(1, 1)$ and $(3, 5)$.

pt (x_1, y_1)
slope m

$$y - y_1 = m(x - x_1) \rightarrow y = mx + b$$

?
0 6a) Solve: $3|x + 1| - 2 = 4$

6b) Solve: $|2x + 1| = |3 - 4x|$

~~7) You have 120 feet of fencing to make a rectangular pen. Find the dimensions of the pen with maximum area.~~

?
0 8a) Solve: $3|x + 1| - 2 > 4$

8b) Solve: $x^2 + 2x \leq x + 6$

?
0 9) For $f(x) = -2(x - 2)^3(x + 1)^7(3 - x)$ what are the zeros of the function with their multiplicities? What is the lead term? What is the end-point behavior of the graph of $f(x)$?

~~10) Two zeros of $f(x) = x^4 + 2x^3 - 7x^2 - 20x - 12$ are $x = 3$ and $x = -1$. Use ~~synthetic division~~ to factor $f(x)$.~~

11) Use long division to divide $p(x) = x^4 - x^3 + 2x^2 - 4x - 8$ by $d(x) = x^2 + 4$. Use the result to factor $p(x)$.

One prob on final uses long division

12) One zero of $p(x) = x^4 - 2x^3 + 26x^2 - 50x + 25$ is $x = 5i$. Use long division to factor $p(x)$.

we add rational, exponential, and log functions to polynomials.

EXAM 2

1) Find the domain and sketch the graph for:

one sketch of rational function. $f(x) = \frac{3x}{x^2 - 4}$

2) Find the domain and sketch the graph for:

$$f(x) = \frac{(x-2)(x-1)}{(x+1)(x-1)}$$

3) Solve the equation:

$$\frac{1}{x} + \frac{1}{x^2 - 1} = 3$$

4) Solve the equation:

$$\frac{x^3 - 2x + 1}{x - 1} = \frac{x - 2}{2}$$

5) Solve the inequality:

$$\frac{x^2 - x}{x - 2} \geq \frac{6}{x - 2}$$

6) For $f(x) = x^2 - x$ and $g(x) = x^2 + x$ find:

a) $(g \circ f)(3)$

b) $(g \circ f)(x)$ and find its domain

practice bc basically same.

7) For $f(x) = x^2 + 2x$ and $g(x) = \sqrt{x - 2}$ find $(f \circ g)(x)$ and its domain.

8) Verify that the given function is one-to-one and then find its inverse.

$$f(x) = \frac{2x}{1 - x}$$

9) Simplify the given expressions:

a) Express the given exponential expression as a single exponential.

$$\frac{3^{x+1}}{3^x} \cdot 9^{x-1}$$

b) Express the given logarithmic expression as a single logarithm.

$$\log(x - 2) + \log(x) - \log(x^2 - 4) + 2 \log(x + 2)$$

10) Solve: $2^{3x} = 16^{1-x}$

11) Solve: $\log_7(1 - 2x) = 1 - \log_7(3 - x)$

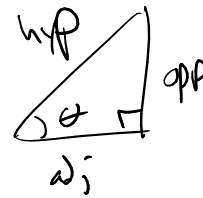
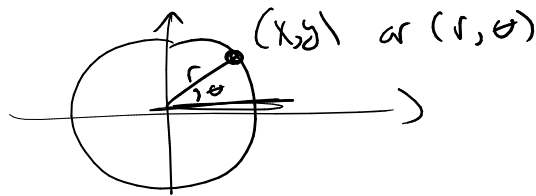
12) Solve: $(\log_2(x))^2 < 2 \log_2(x) + 3$

Rational
Solve
Egn's
1) factor
2) Mult. by common denom.
→ this is now a poly. eqn

11) $\log \frac{(1-2x)(3-x)}{(1-2x)(3+x)} = 1$
→ $(1-2x)(3-x) = 7$
 $2^x = 3^{x^2 - 1}$

$\ln(2) = \ln(3^{x^2-1})$
 $\times \ln 2 = (x^2-1) \ln 3$
etc

EXAM 3



1) Convert the following angles into radians and plot them.

- a) 20°
- b) 421°
- c) -379°

2) Draw the unit circle with all typical angles, label its points, and create the table of values for $\sin(\theta)$, $\cos(\theta)$, and $\tan(\theta)$ for Quadrant I.

3) Find all angles for the following:

- a) $\sin(\theta) = 1/2$
 - b) $\cos(\theta) = \sqrt{3}/2$
 - c) $\sin(\theta) = -\sqrt{2}/2$
- \uparrow \uparrow

4) Find the exact value for the following:

- a) $\csc(\pi/4)$
 - b) $\cot(-7\pi/6)$
 - c) $\cos(14\pi/3)$
 - d) $\sin(\pi/12) =$
- \uparrow

5) Find all angles for the following:

- a) $\cot(\theta) = \sqrt{3}/3$
- b) $\tan(\theta) = -\sqrt{3}$
- c) $\sin(2\theta) = -1/2$

6) Given the following trig function find the values of the other five trigonometric functions.

- a) $\cos(\theta) = -3/5$ with θ in Quadrant II
 - b) $\cot(\theta) = \sqrt{5}$ with θ in Quadrant III
- \uparrow \uparrow

7) Simplify the following expressions:

a) $\frac{\csc(\theta)}{1 + \cot^2(\theta)}$ and write your answer using only $\sin(\theta)$

b) Verify that $\cos(4\theta) - 4\cos(2\theta) + 3$ simplifies to $8\sin^4(\theta)$

c) Write $\sec^2(\theta)\tan(\theta) - \cot(\theta)$ using only $\sin(\theta)$ and $\cos(\theta)$.

8) Graph: $f(x) = 2\sin(\pi x) + 2$

9) Graph: $f(x) = 3\sec(2\pi x)$

10) Find the exact values:

a) $\arcsin(1/2)$

b) $\arccos(-1/2)$

c) $\arctan(-\sqrt{3})$

11) Find the exact values:

a) $\sin(\arccos(-1/2))$

b) $\cot(\arcsin(12/13))$

12) Solve: $4\sin(x)\cos(x) + 2\sin(x) = 1 + 2\cos(x)$

1x

$\sin(2x) + \sin(x) = 0$

EXAM 4

1) Standing beside a river you notice a leaf floating on the water is moving up and down with the waves. Assuming that the motion is sinusoidal, find a function for its motion if it takes 2 seconds to go through one oscillation, the river level (when still) is 15 feet, and the range from peak to valley of the motion is 6 inches. Include a drawing for the observations.

Problems 2) to 5) Find all angles and sides to given triangles.

6) Convert the polar coordinates into (x, y) coordinates.

a) $r = 3$ and $\theta = \pi/4$ (find the exact x and y values)

b) $r = 1.2$ and $\theta = 3.01$ (do not find exact x and y values). What quadrant is the point in?

7) Convert the (x, y) coordinates into polar coordinates.

a) $x = 1$ and $y = 3$ (do not find exact values for r and θ)

b) $x = -2$ and $y = 2$ (do not find exact values for r and θ)

8) Solve the system using substitution. Note: You should find the solution $(1,2,3)$.

$$\begin{aligned}x + y - z &= 0 \\2x - y + z &= 3 \\-3x + y + z &= 2\end{aligned}$$

9) Solve the system using elimination. Note: You should find the solution $(1,2,3)$.

$$\begin{aligned}x + y - z &= 0 \\2x - y + z &= 3 \\-3x + y + z &= 2\end{aligned}$$

10) Solve the system using an augmented matrix. Note: You should find the solution $(1,2,3)$.

$$\begin{aligned}x + y - z &= 0 \\2x - y + z &= 3 \\-3x + y + z &= 2\end{aligned}$$

11) Perform the given matrix operations.

$$\begin{pmatrix} -1 & 1 & 0 \\ 2 & 3 & -1 \end{pmatrix} \begin{pmatrix} -1 & 2 \\ 1 & -1 \\ -1 & 1 \end{pmatrix} - 2 \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}$$

12) Verify that A and B are inverses by using multiplication.

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 2 & -2 \\ 2 & 5 & -1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 8 & -3 & -2 \\ -3 & 1 & 1 \\ 1 & -1 & 0 \end{pmatrix}$$