


Math 112

Q's

$$\sum_{k=1}^n k$$

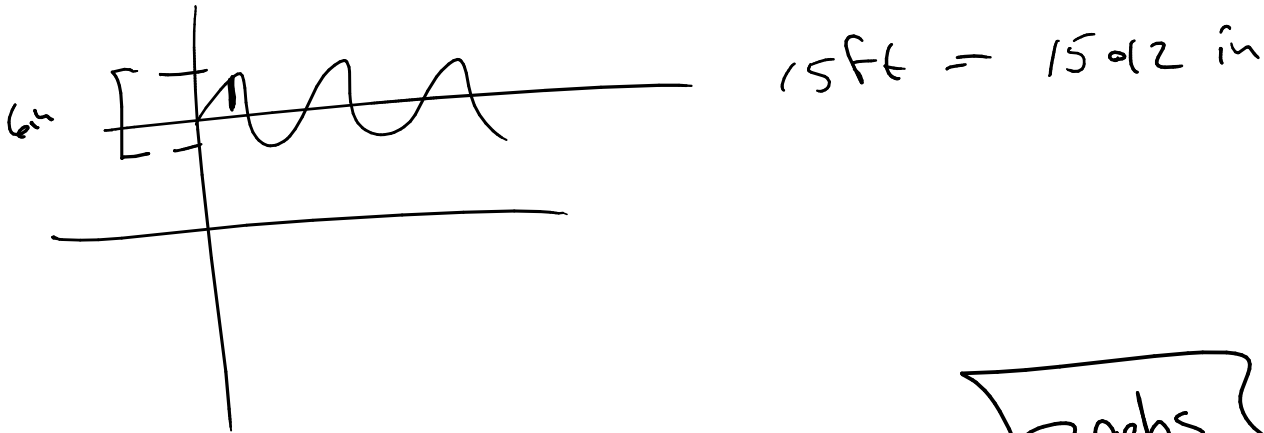
$$\sum_{k=1}^n k = \frac{n(n+1)}{2}$$

Final Review
Continued

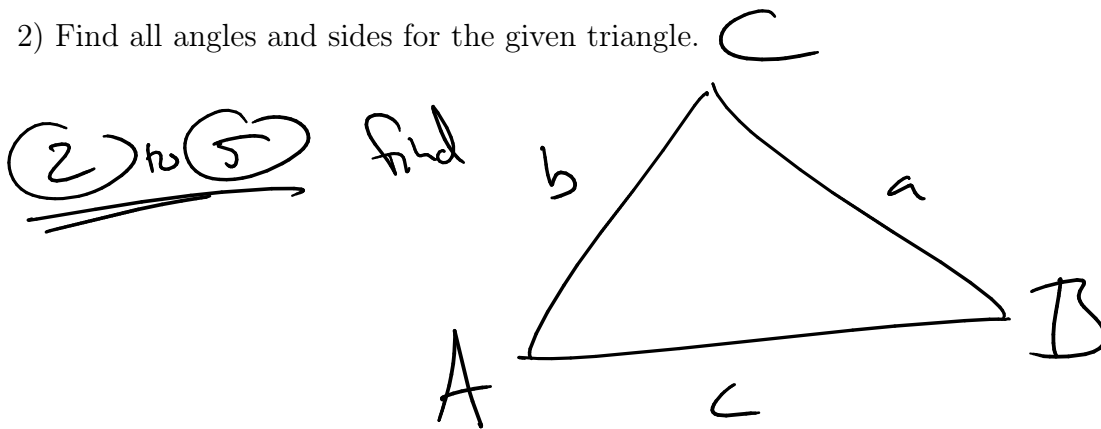


EXAM 4

1) Standing beside a river you notice a leaf floating on the water is moving up and down with the waves. Assuming that the motion is sinusoidal, find a function for its motion if it takes 2 seconds to go through one oscillation, the river level (when still) is 15 feet, and the range from peak to valley of the motion is 6 inches. Include a drawing for the observations.



2) Find all angles and sides for the given triangle.



① Use Law of Sines

② Use Law of Cosines

✓ (a) $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

✓ (b) $a^2 = b^2 + c^2 - 2bc \cos A$

5) Find all angles and sides for the given triangle.

6) Convert the polar coordinates into (x, y) coordinates.

a) $r = 3$ and $\theta = \pi/4$ (find the exact x and y values)

b) $r = 1.2$ and $\theta = 3.01$ (do not find exact x and y values). What quadrant is the point in?

7) Convert the (x, y) coordinates into polar coordinates.

a) $x = 1$ and $y = 3$ (do not find exact values for r and θ)

b) $x = -2$ and $y = 2$ (do not find exact values for r and θ)

8) Solve the system using substitution. Note: You should find the solution (1,2,3).

9) Sub.
10) Elim
11) Matrix

$$\begin{aligned}x + y - z &= 0 \\2x - y + z &= 3 \\-3x + y + z &= 2\end{aligned}$$

ex

$$\begin{cases}x + y = 3 \\x - 2y = -3\end{cases} \quad \text{2x2}$$

1 prob

Solve by any method

11) Perform the given matrix operations.

$$\begin{pmatrix} -1 & 1 & 0 \\ 2 & 3 & -1 \end{pmatrix} \begin{pmatrix} -1 & 2 \\ 1 & -1 \\ -1 & 1 \end{pmatrix} - 2 \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}$$

① ~~Prob~~
Matrix Ops

12) Verify that A and B are inverses by using multiplication.

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 2 & -2 \\ 2 & 5 & -1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 8 & -3 & -2 \\ -3 & 1 & 1 \\ 1 & -1 & 0 \end{pmatrix}$$

EXAM 5

1) Write the first five terms of the given sequence.

a) $\{2^n + 1\}_{n=-2}^{\infty}$ $2^{-2} + 1$, $2^{-1} + 1$, etc
 $n = -2$ $n = -1$

b) $\{2 \cdot (\frac{1}{3})^{k+1}\}_{k=0}^{\infty}$ $2(\frac{1}{3})^0$, etc
 $k = 0$

c) $a_0 = 1, a_1 = 2$ and $a_n = a_{n-1} - 2a_{n-2}$ 1, 2, 0, -1, -1, ...

2) Find both a closed and open formula for the n^{th} term of the given sequence.

a) 3, 6, 11, 18, 27, ... Squares 1, 4, 9, 16, 25, 36, --
Sub. $\sqrt{3}$ $\sqrt{6}$ $\sqrt{11}$ $\sqrt{18}$ $\sqrt{27}$..

$a_0 = 3$ $a_n = a_{n-1} + (\text{add's})_{n+1}$ $n = 2$

b) $\frac{2}{3}, 2, 6, 18, 54, 162, \dots$
Div. $\sqrt{\frac{2}{3}}$ $\sqrt{2}$ $\sqrt{6}$ $\sqrt{18}$ $\sqrt{54}$ $\sqrt{162}$
 $= 3$

c) 5, 7, 10, 14, 19, ... •, ••, •••, ••••, •••••
Sub. $\sqrt{5}$ $\sqrt{7}$ $\sqrt{10}$ $\sqrt{14}$ $\sqrt{19}$..
Triangular 1 3 6 10
 \downarrow \downarrow \downarrow \downarrow
 +2 +3 +4 +5

3) Find the sum. Do not simplify your answer.

a) $\sum_{k=1}^{99} 2k^3 - 5$

$$= \sum_{k=1}^{99} 2k^3 - 5 = 2 \left[\sum_{k=1}^{99} k^3 \right] - \left[\sum_{k=1}^{99} 5 \right]$$

$$\sum_{k=1}^n k^3 = \left(\frac{n(n+1)}{2} \right)^2$$

$$= \left(2 \left(\frac{99(100)}{2} \right) - 5 \cdot 99 \right)$$

b) $\sum_{k=0}^{99} 5^k$

$$\sum_{k=0}^n ar^k = a \left(\frac{r^{n+1} - 1}{r - 1} \right)$$

$$\sum_{k=0}^{99} (1)(5)^k$$

4) Find the sum. Do not simplify your answer.

a) $\sum_{k=19}^{99} (k^3 - k^2 + k + 2)$

Use

$$\sum_{k=1}^{99} a_k - \sum_{k=1}^{18} a_k = \sum_{k=19}^{99} a_k$$

b) $\sum_{k=3}^{99} 2k - 5^{k-1}$

$$\sum_{k=3}^{99} 2k - \sum_{k=3}^{99} 5^{k-1}$$

$$\sum_{k=3}^{99} 5^{k-1} = \sum_{k=3}^{99} \frac{1}{5} 5^k = \sum_{k=0}^{99} \left(\frac{1}{5} \right) 5^k - \sum_{k=0}^2 \frac{1}{5} \cdot 5^k$$

etc

5) Expand the given binomial.

a) Use Pascal's Triangle to expand $(x - x^{-2})^4$

$$= (x)^4 - 4(x)^3(x^{-2}) + 6(x)^2(x^{-2})^2 - 4(x)(x^{-2})^3 + (x^{-2})^4$$



= simplify.



b) Use the Binomial Theorem to expand $(2x + x^{-1})^4$.

6) Use the Binomial Theorem to find the indicated term. Leave your coefficients in factorial notation.

a) The term containing x^{17} in the expansion of $(x + 2y)^{23}$.

$$x^{23} + 23x^{22}(2y) + \dots + \frac{23!}{17!6!} (x)^{17} (2y)^6 + \dots$$

$$\frac{23!}{17!6!} 2^6 x^{17} y^6$$

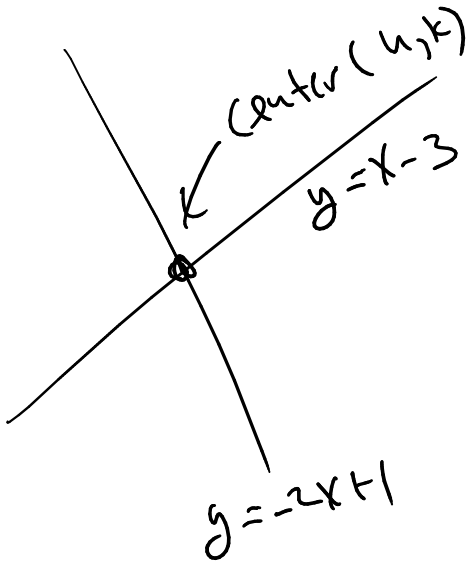
b) The term containing x and y with the same power in the expansion of $(2x - y)^{24}$.

$$\frac{24!}{12!12!} (2x)^{12} (-y)^{12}$$

$$= 2^{12}$$

Note: 7 to 11 are centers. (1 prob) recog. the eqns

7) Write the standard equation of the circle and then graph it. Where the center is the intersection of $2x + y = 1$ and $x - y = 3$ and the radius is 4.



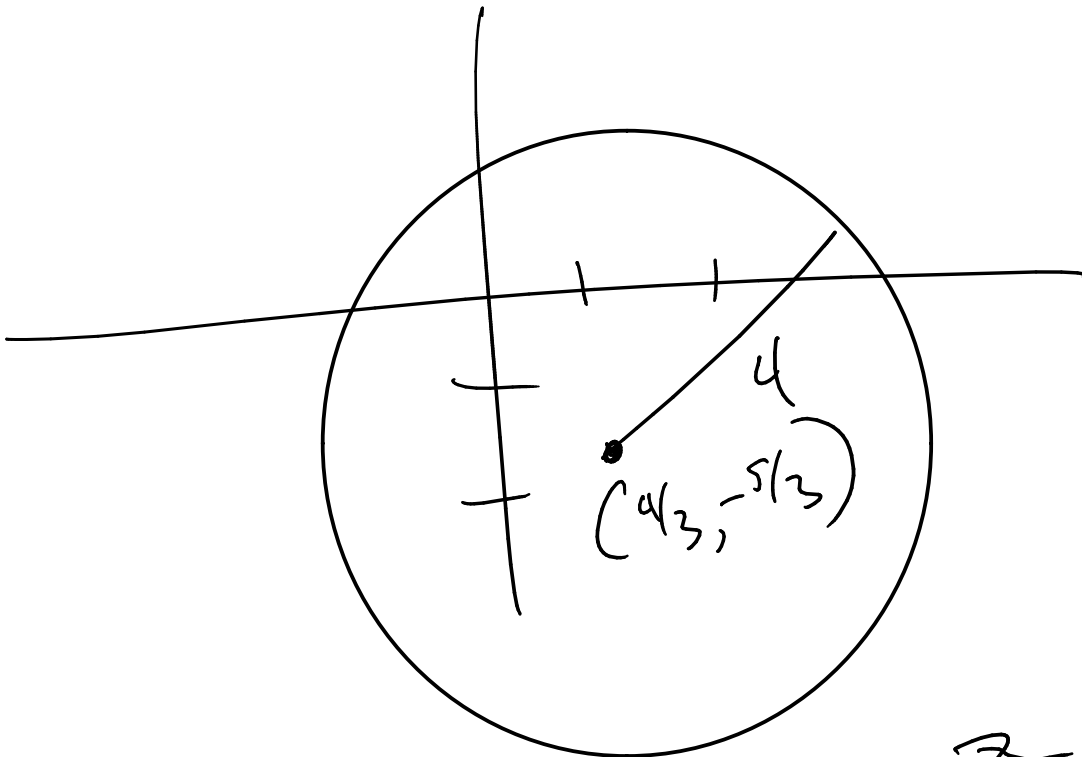
$$\begin{cases} 2x + y = 1 \\ x - y = 3 \end{cases}$$

$$r = 4$$

$$3x = 4$$

$$x = \frac{4}{3} = h$$

$$y = \frac{4}{3} - 3 = -\frac{5}{3} = k$$



$$(x - \frac{4}{3})^2 + (y + \frac{5}{3})^2 = 4^2$$

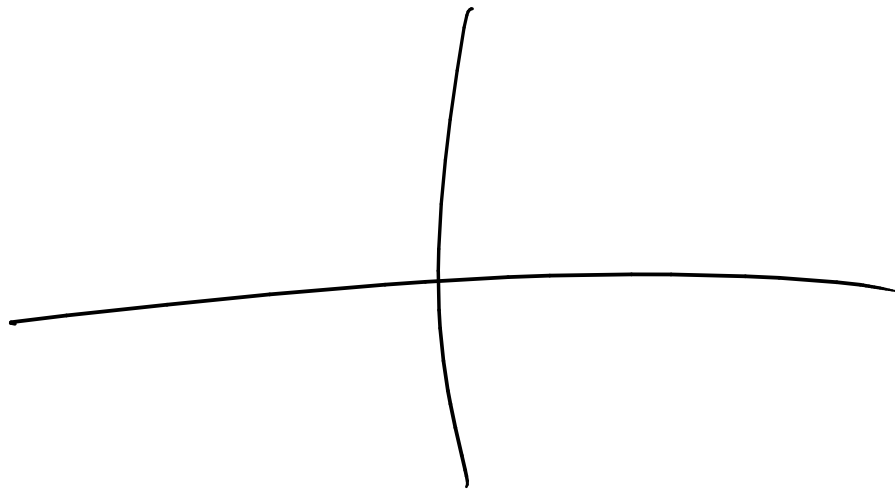
8) Put the circle $x^2 + y^2 - 6x + 4y + 9 = 0$ in standard form and graph it.

$$x^2 - 6x + (-3)^2 + y^2 + 4y + (2)^2 = \underline{-9} + \underline{(-3)^2} + (2)^2$$

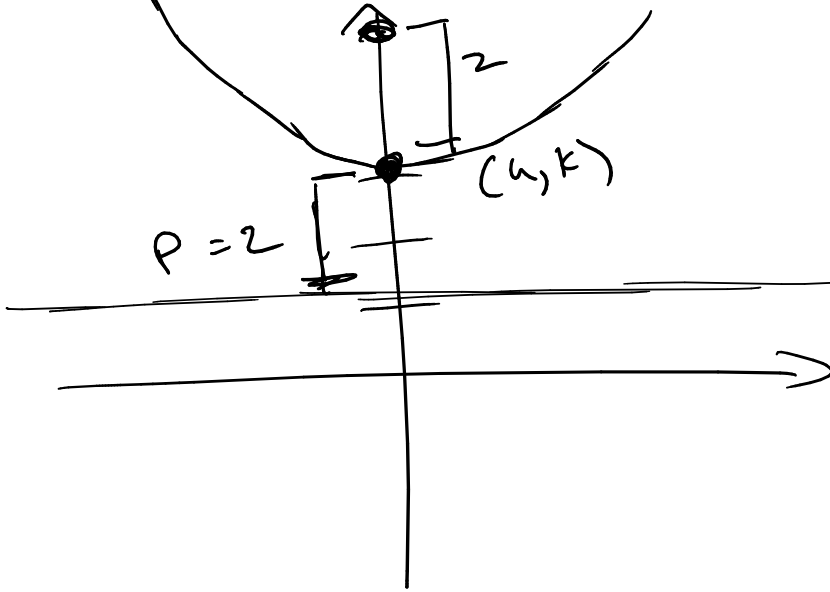
$$(x-3)^2 + (y+2)^2 = 4$$

center $(3, -2)$

$$r = 2$$



9) Find an equation for the parabola whose vertex is at the y -intercept of $x + 2y = 6$ and directrix is the line $y = 1$ and graph it. Include the focus in your graph.



$$y = -\frac{1}{2}x + 3$$

ok

10) Put the ellipse $4x^2 - 16x + 9y^2 + 18y - 11 = 0$ in standard form and graph it. Include the center, major axis, minor axis, endpoints of each axis, and the foci in your graph.

$$4x^2 - 16x + 9y^2 + 18y = 11$$

$$4 \left[x^2 - 4x + (-2)^2 \right] + 9 \left[y^2 + 2y + (1)^2 \right] = 11 + 16 + 9$$

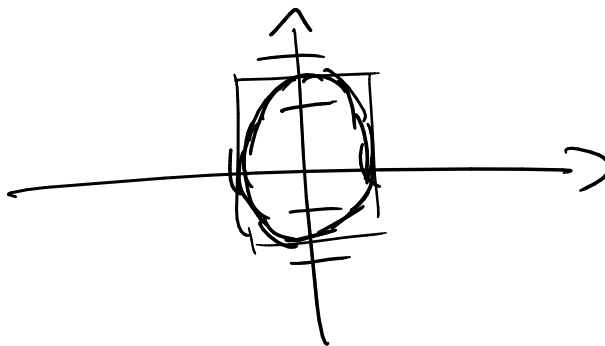
finish
~

11) Graph the hyperbola $\frac{(x+2)^2}{9} - \frac{(y-2)^2}{4} = 1$. Include the center, transverse axis, conjugate axis, vertices, foci, and asymptotes in your graph.

(ex)

$$x^2 + \frac{y^2}{3} = 1$$

ellipse



$$\frac{(x+1)^2}{4} - y^2 = 1$$

