Math 415

ToDo (Weekend) $\boxed{\text{Read}}$ Ch1,2 of Math Reasoning.

Logic

Declare

Given a set of objects $\equiv$ Univ. of Discourse.

U.D.

ex My name is Mark, I am blue

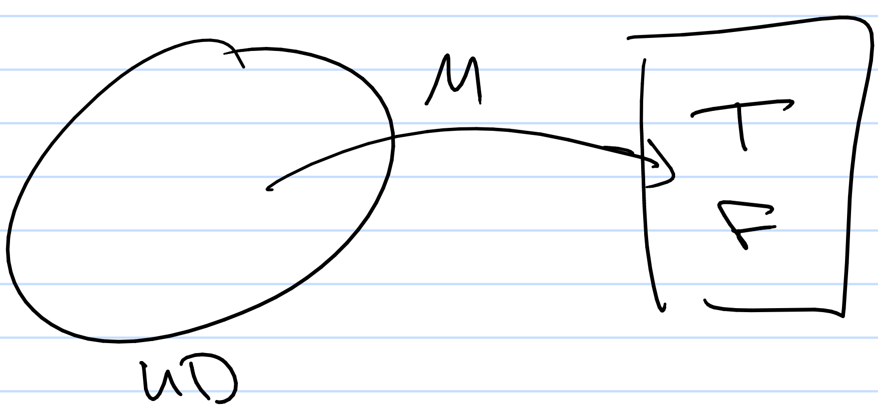
M : "My name is Mark"

blue : "I am blue"

vs $M(o)$: "o is named Mark"

where o is from the set of All People

U.D.



p	q	$\neg p$	$\neg q$	$p \wedge q$	$p \vee q$	$p \oplus q$	$p \to q$	$p \leftrightarrow q$
T	T	F	F	T	T	F	T	T
T	F	F	T	F	T	T	F	F
F	T	T	F	F	T	T	T	F
F	F	T	T	F	F	F	T	T

eng ⇄ $\boxed{\text{sym}}$

→ $\boxed{\text{Logically Equiv}}$ (Same)

Consider: $p \to q \Leftrightarrow \neg p \vee q$

p	q	$p \to q$	$\neg p$	$\neg p \vee q$	$(p \to q) \leftrightarrow (\neg p \vee q)$
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

all true = tautology!

use $(p \to q) \equiv (\neg p \vee q)$

p: x is an even number

q: x^2 is an even number

(If x is even, then x^2 is even)

≡ (x is odd or x^2 is even)

Logical Equiv.

① $(p \to q) \equiv \neg p \vee q$ disjunctive version of $p \to q$

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② $(p \to q) \equiv \neg q \to \neg p$ contra positive version of $p \to q$

if n^2 is even, then n is even.

$\equiv$ if n is odd, then n^2 is odd.

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③ $p \vee T \equiv T$ $p \wedge F \equiv F$

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④ $p \vee F \equiv p$ $p \wedge T \equiv p$

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⑤ $p \vee \neg p \equiv T$ $p \wedge \neg p \equiv F$

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⑥ $p \vee p \equiv p$ $p \wedge p \equiv p$

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⑦ $\neg(\neg p) \equiv p$

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⑧ $p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$

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(9) $(p\lor q)\lor r \equiv p\lor(q\lor r)$

$(p\land q)\land r \equiv p\land(q\land r)$

(10) $p\land(q\lor r) \equiv (p\land q)\lor(p\land r)$

$p\lor(q\land r) \equiv (p\lor q)\land(p\lor r)$

(11) $\neg(p\land q) \equiv \neg p\lor\neg q$

$\neg(p\lor q) \equiv \neg p\land\neg q$

(12) $(p\leftrightarrow q) \equiv (p\to q)\land(q\to p)$

(13) $(p\leftrightarrow q) \equiv (\neg p\leftrightarrow\neg q)$

Quantification

What about $P(x)$: "x has a property P"
x is from a U.D.

$Q(x)$: "x has a property Q"
x is from a U.D.

Making a propositional function into a proposition is Quantification.

① evaluate: ex $E(x)$; "x is an even integer"

$E(3)$: $\boxed{\text{"3 is an even integer"}} \equiv F$

② Universal Quantification: $\forall x\, P(x)$

$\equiv$ "for all x in the U.D. P(x) is true"

③ Existential Quantification $\exists x\, P(x)$

$\equiv$ "there exists some x in the U.D such that P(x) is true."

Logical Equiv.

① $\neg \forall x\, P(x) \equiv \exists x\, \neg P(x)$

② $\neg \exists x\, P(x) \equiv \forall x\, \neg P(x)$

if U.D. is $o_1, o_2, o_3, \ldots, o_n$ (finite)

③ $\forall x\, P(x) \equiv (P(o_1) \wedge P(o_2) \wedge \cdots \wedge P(o_n))$

④ $\exists x\, P(x) \equiv (P(o_1) \vee P(o_2) \vee \cdots \vee P(o_n))$

All ravens are black.

Monday:

raven: "$\qquad$"

black: "$\qquad$"

connectives

$R(x)$: "x is a raven"

$B(x)$: "x is black"

connective + quantifiers