Math 415

Due Friday Mathematical Reasoning

2.1 (1, 2, 3, 5, 7, 11, 12)
2.2 (1ac, 2ac, 3ac, 4a, 7, 9ad, 10)

Q All ravens are black. (Paradox)

Set theory / Quantification / logical Equivalences.

"All ravens are black." → symbols?

Attempts? ?① $\forall x\,(R(x)) \wedge \forall x\,(B(x))$ ←

?② $R(x) \wedge B(x)$ ←

?③ if raven, then Black $x \to y$

x: "raven"
y: "black"

(x) $\forall y\; P(y) \to \exists y\; Q(y)$

$\forall x\, R(x) \land \forall x\, B(x)$

R(x): "x is a raven." — U.D = Set of all raven

B(x): "x is black."

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if raven, then black seems ok-ish.

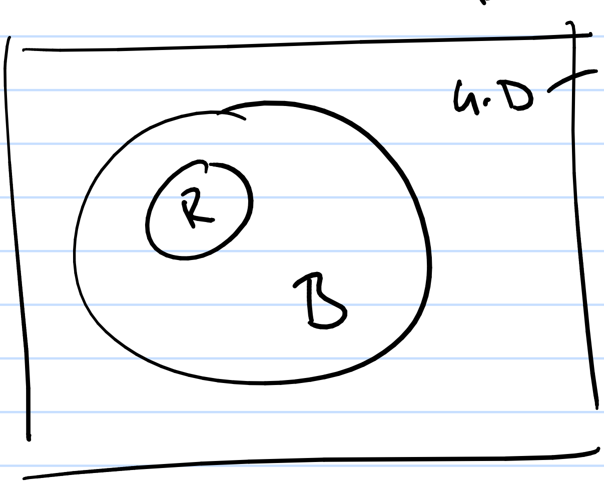
U.D. is all ravens.

B(x): "x is black" U.D is

All ravens are black is $\boxed{\forall x\, B(x)}$

U.D. of all birds

$$\forall b\,(R(b) \to B(b))$$

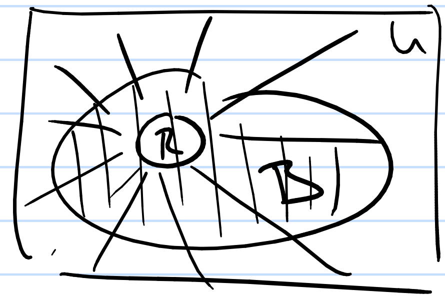


all birds

$$R \subseteq B$$

$$= \{e \mid e \in R \to e \in B\}$$

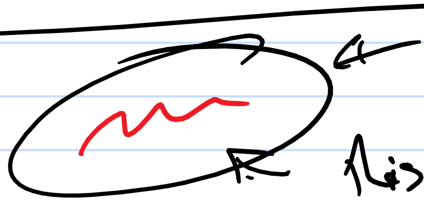
All ravens are black.



$$\forall x\,(\text{Raven}(x) \to \text{Black}(x))$$
$$\equiv \forall x\,(\neg \text{Raven}(x) \vee \text{Black}(x))$$
$$\equiv \forall x\, \neg (\text{Raven}(x) \wedge \neg \text{Black}(x))$$
$$\equiv \neg \boxed{\exists x\, (\text{Raven}(x) \wedge \neg \text{Black}(x))}$$

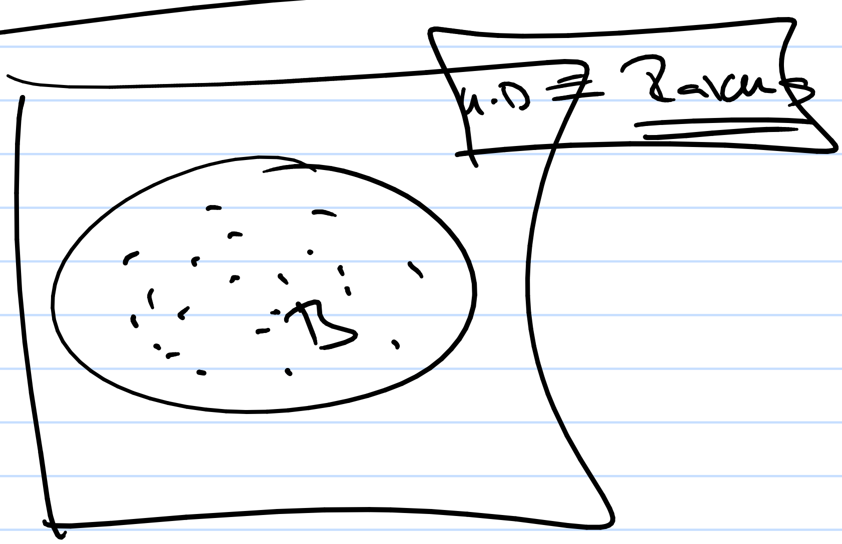
R	B	
T	T	T
T	F	F
F	T	T
F	F	T

counter example to the "All" statement

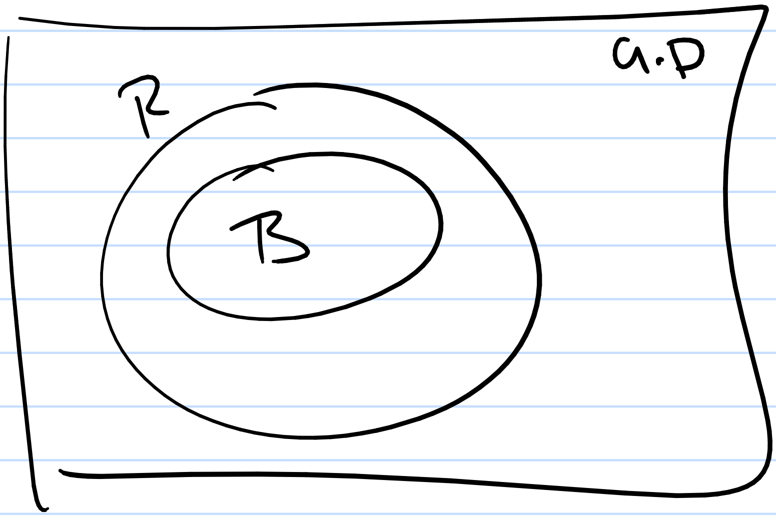


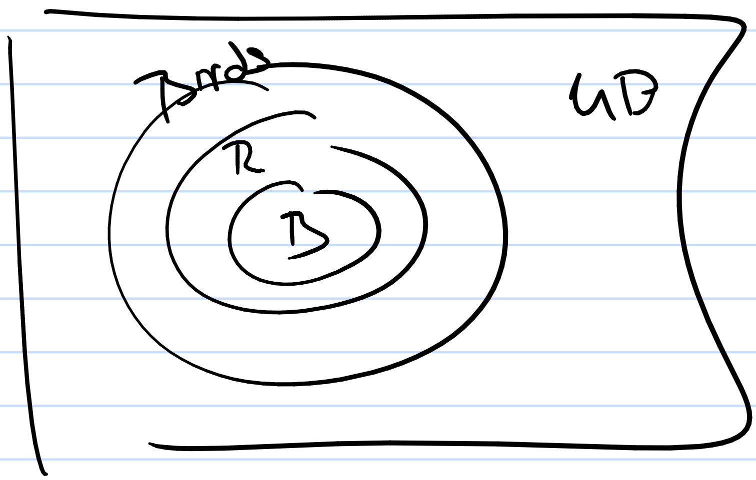
This red squiggle is not a counter example!

So by logic All ravens are black.

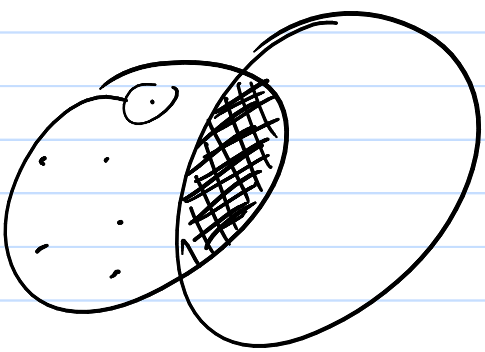


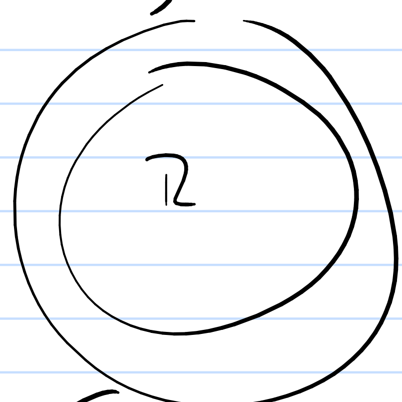
$$\forall x\,(B(x))$$





$\neg\ \exists x\ (\cancel{\mathrm{Bird}}(x) \wedge \mathrm{Raven}(x) \wedge \neg \mathrm{Black})$





Counter example:

All ravens are black

$$\forall x\,(\text{Raven}(x) \to \text{black}(x))$$

$$\equiv \neg \exists x\,(\text{Raven}(x) \wedge \neg \text{black}(x))$$

but if my pet modified raven **Joe** is a pink raven

$$\text{Raven}(\text{Joe}) \wedge \neg \text{black}(\text{Joe}) \equiv T$$

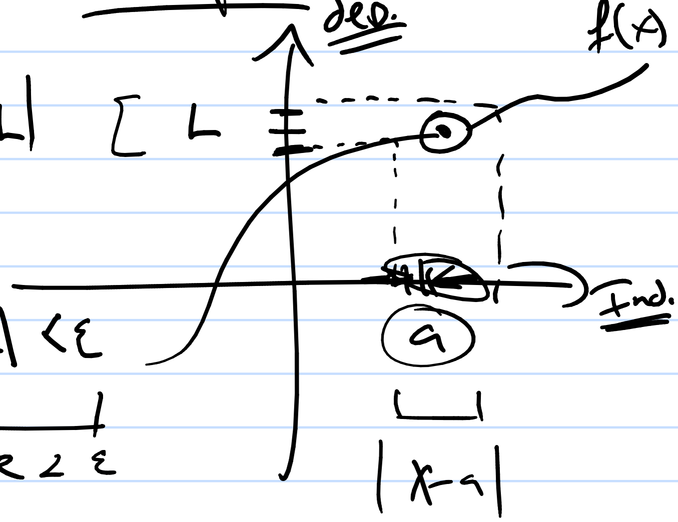
So All ravens are black is false.

Nested Quantification

$$\lim_{x \to a} f(x) = L$$

ε-δ proof

$|f(x)-L|$



$\forall \varepsilon > 0$ **$\exists \delta > 0$**

$$\text{if } \underbrace{|x-a| < \delta}_{\text{distance} < \delta} \to \underbrace{|f(x)-L| < \varepsilon}_{\text{distance} < \varepsilon}$$

Abilities

1. symbols, ops $\rightleftarrows$ eng
2. show tautologies by truth tables
3. show logical equiv. by truth tables
4. use logical equiv.

$$\neg(r \to q) \equiv \neg(\neg r \vee q) \equiv \neg\neg r \wedge \neg q$$
$$\equiv r \wedge \neg q$$

Rules of Inference (tautologies)

← used in mathematical arguments.

$$(\text{proposition}1 \wedge \text{proposition}2 \wedge \dots \wedge \text{proposition}\, n) \to \text{conclusion}$$

Valid: means the argument ↑ is a tautology.

Notation

proposition 1
proposition 2
⋮
proposition n

$\therefore$ conclusion

horse

Rule of Inference (valid argument forms)

$p \to q$
p
$\therefore q$

Affirming the hypothesis.

$$((p \to q) \land p) \to q \overset{?}{\equiv} T$$

$$\equiv \neg((p \to q) \land p) \lor q$$

$$\equiv (\neg(p \to q) \lor \neg p) \lor q$$

$$\equiv \neg(p \to q) \lor (\neg p \lor q)$$

$$\equiv \neg(p \to q) \lor (p \to q)$$

$$\equiv T$$