Math 415

Due Tues 2.3 (1ac, 2, 3)
2.4 (1, 2bc, 3ac, 4abcd, 11*)

extra credit

Q's

* **2.** Suppose that P and Q are statements for which $P \to Q$ is true and for which $\neg Q$ is true. What conclusion (if any) can be made about the truth value of each of the following statements?

(a) P **(b)** $P \wedge Q$ **(c)** $P \vee Q$

P	Q	$\neg Q$	$P \to Q$
T	T	F	T ←
T	F	T	F ←
F	T	F	T ←
F	F	T	T ← ←

* **7.** Use previously proven logical equivalencies to prove each of the following logical equivalencies about **conditionals with conjunctions:**

(a) $[(P \wedge Q) \to R] \equiv (P \to R) \vee (Q \to R)$

(b) $[P \to (Q \wedge R)] \equiv (P \to Q) \wedge (P \to R)$

Table?

P	Q	R	. . . - - . .	$((P \wedge Q) \to R) \leftrightarrow ((P \to R) \vee (Q \to R))$
T	T	T		T
T	T	F		T
T	F	T		T
T	F	F		T
F	T	T		T
F	T	F		T
F	F	T		T
F	F	F		T

Use log. equiv?. (Similar to algebra: "simplify")

left $\equiv$ step1 $\equiv$ step2 $\equiv$ ... $\equiv$ right

* **7.** Use previously proven logical equivalencies to prove each of the following logical equivalencies about **conditionals with conjunctions:**

(a) $[(P \wedge Q) \to R] \equiv (P \to R) \vee (Q \to R)$

(b) $[P \to (Q \wedge R)] \equiv (P \to Q) \wedge (P \to R)$

$(P \wedge Q) \to R$

$\equiv \neg(P \wedge Q) \vee R$

$\equiv \neg P \vee \boxed{\neg Q \vee R}$

$\equiv Q \to R$

$(P \to R) \vee (Q \to R)$

$\equiv (\neg P \vee R) \vee (\neg Q \vee R)$

$\equiv \neg P \vee \neg Q \vee R \vee R$

$\equiv \neg P \vee \neg Q \vee R$

Hence in $(P \wedge Q) \to R \equiv \neg(P \wedge Q) \vee R \equiv (\neg P \vee \neg Q) \vee R$

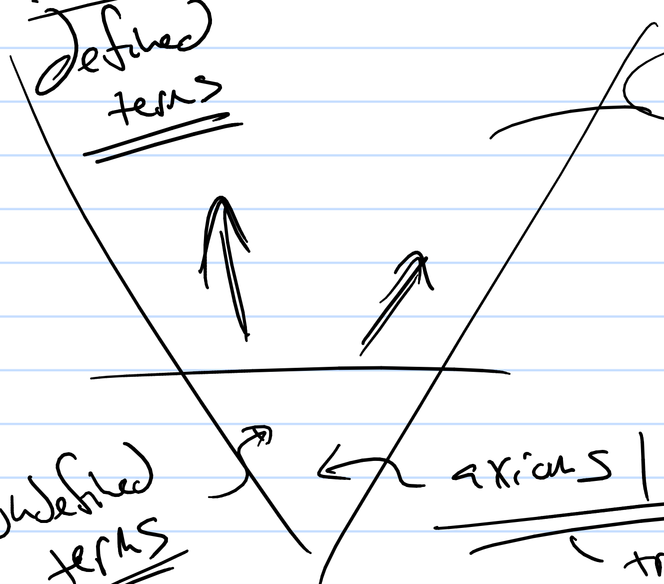
$\equiv \neg P \vee \neg Q \vee R \vee R \equiv \neg P \vee R \vee \neg Q \vee R$

$\equiv (P \to R) \vee (Q \to R)$

Arguments!

Axiomatic Method

Valid Mathematical argument



defined terms

statements we can prove to be true

undefined terms

axioms / postulates

true statement that is just plain true

Valid Argument (useful tautology)

$\boxed{\text{Valid Argument Form}}$ is a tautology of variables

$$\underbrace{[(\text{premise } 1) \land (\text{premise } 2) \land \cdots \land (\text{premise } n)] \to \text{Conclusion}}_{\text{argument}}$$

Valid means $\boxed{\text{true}}$

∃ : such that

∴ : hence

■ qed

premise 1
premise 2
⋮
premise n

∴ conclusion

must be true

$\boxed{\text{ex}}$

$$\begin{array}{l} (P \land Q) \to R \\ \text{Law of disjunction implication} \\ \hline \therefore \neg(p \land q) \lor R \end{array}$$

$(\square \to \Delta) \equiv \neg\square \lor \Delta$

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore q \end{array}$$

affirming the hypothesis or Modus ponens

$$\begin{array}{l} p \to q \\ \neg q \\ \hline \therefore \neg p \end{array}$$

denying the conclusion or modus tollens

Note: $(p \to q) \equiv \neg q \to \neg p$

$$\begin{array}{l} \neg q \to \neg p \\ \neg q \\ \hline \therefore \neg p \end{array}$$

$$\begin{array}{l} p \to q \\ \underline{q \to r} \\ \therefore p \to r \end{array}$$

hypothetical syllogism

$$\begin{array}{l} p \lor r \\ \underline{\neg p} \\ \therefore r \end{array}$$

disjunctive syllogism

$$\begin{array}{l} \underline{p} \\ \therefore p \lor r \end{array}$$

Addition

$$\begin{array}{l} \underline{p \land r} \\ \therefore p \end{array}$$

Simplification

$$\begin{array}{l} p \lor q \\ \underline{\neg p \lor r} \\ \therefore q \lor r \end{array}$$

resolution

$$\begin{array}{l} \underline{\forall x \, P(x)} \\ \therefore P(c) \text{ for any } c \end{array}$$

Universal Instantiation

$$\begin{array}{l} \underline{\text{show for any } c \;\; P(c) \text{ is true}} \\ \therefore \forall x \, P(x) \end{array}$$

Universal Generalization

$$\frac{\exists x\, P(x)}{\therefore P(c) \text{ for one (or more) specific } c}$$

Existential Instantiation

$$\frac{P(c) \text{ is true for some } c}{\therefore \exists x\, P(x)}$$

Existential Generalization

(☆) If superman were able and willing to prevent evil, he would do so.
If superman were unable to prevent evil, he would be impotent.
If superman were unwilling to prevent evil, he would be malevolent.
Superman does not prevent evil.
If superman exists, he is neither malevolent nor impotent.

premise

therefore, Superman does not exist. conclusion

from Kalish and Montague

Able : "Superman is able to prevent evil"
Will : " " " willing " " "
Prevent : "Superman prevents evil"

$(\text{Able} \land \text{Will}) \to \text{Prevent}$
$\neg\text{Able} \to \text{Impotent}$
$\neg\text{willing} \to \text{Malevolent}$
$\neg\text{Prevent}$
$\text{exists} \to (\neg\text{Malevolent and } \neg\text{impotent})$

$\therefore \neg\text{exist}$