Math 415

Q5

p	q	$\neg p$	$\neg q$	$p \wedge q$	$p \vee q$	$p \oplus q$	$p \to q$	$p \leftrightarrow q$
T	T	F	F	T	T	F	T	T
T	F	F	T	F	T	T	F	F
F	T	T	F	F	T	T	T	F
F	F	T	T	F	F	F	T	T

$$[((p \to q) \wedge p) \Rightarrow q] \equiv T$$

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore\ q \end{array}$$

$(p \to q) \wedge p \equiv T$

$q \equiv F$

* **1.** For each of the following, write the statement as an English sentence and then explain why the statement is false.

(a) $(\exists x \in \mathbb{Q})\left(x^2 - 3x - 7 = 0\right)$.

(b) $(\exists x \in \mathbb{R})\left(x^2 + 1 = 0\right)$. $\longrightarrow$

(c) $(\exists m \in \mathbb{N})\left(m^2 < 1\right)$.

$\exists x\ P(x)$: "there is some x in the U.D. such that $P(x)$ is true"

$\forall x\ P(x)$: "for all x in U.D. $P(x)$ is true."

$\exists x \in \mathbb{R}$

$\mathbb{R}$: "all reals"

$\in$: "element of"

$\to$ "there is some real number x such that ..."

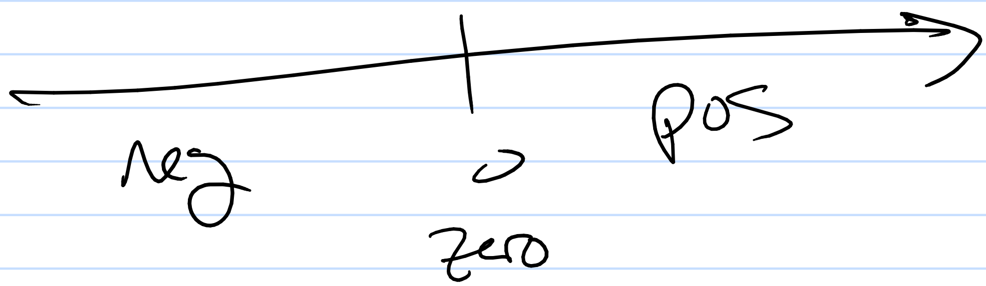
there is a real such that

$$\exists x \in \mathbb{R} \, (x^2 + 1 = 0)$$

$P(x)$: " $x^2 + 1 = 0$ is true"

$$x^2 = -1$$

$$x \cdot x = -1$$



" there is a real number, such that it times itself is -1"

"Some real number squared is minus one."

False?

So you are saying $\rightarrow \exists x \in R \, (x^2 + 1 = 0)$

$$\forall x \in R \, (x^2 \neq -1)$$

is true

$\mathbb{N} = \{1, 2, 3, \dots\}$ $\boxed{\text{positive ints}}$

$\mathbb{W} = \{0, 1, 2, 3, \dots\}$ $\boxed{\text{non-neg ints}}$

---

compare to english   unable ~ ¬able

able vs $\boxed{\neg\text{able}}$ vs $\boxed{\text{unable}}$?

Sets: $\emptyset$ empty set

$\mathbb{N} = \{1, 2, 3, \dots\}$

$\mathbb{W} = \{0, 1, 2, \dots\}$

$\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$

$\mathbb{Z}^+ = \{1, 2, 3, \dots\}$

$\mathbb{Z}^- = \{\dots, -3, -2, -1\}$

Non Neg $= \{0, 1, 2, \dots\}$

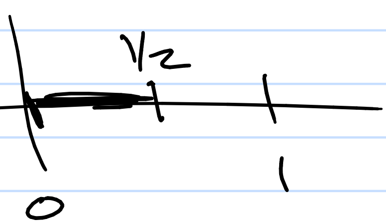
Non Pos $= \{-3, -2, -1, 0\}$

Rationals: $\left\{ \frac{a}{b} \;\middle|\; a \in \mathbb{Z} \wedge b \in \mathbb{Z} \wedge b \neq 0 \right.$ $\wedge$ a,b have no common factor $\}$

such that

0.121221222122221...

irrational



Note: a rational as a decimal will terminate or repeats

ex $\frac{1}{4} = 0.25$

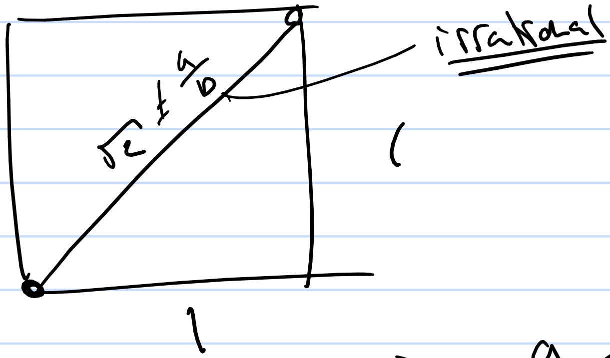
$\frac{1}{9} = 0.1111\dots$

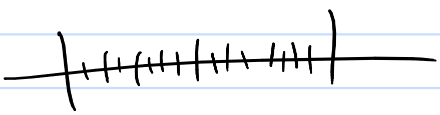
irrationals in decimal do not (term or repeat)

↳ not term and not repeat

irrationals not term and not repeat

ex





$\cancel{\frac{a}{b} =} \boxed{a\left(\frac{1}{b}\right)}$

Reals $\mathbb{R}$ = rationals $\cup$ irrationals

$\underbrace{\text{rationals} \cup \text{irrationals}}_{\text{any decimal}}$

Solve $x^2+1=0$

$x^2=-1$ nope! (no real)

call answer $x=i$ and $x=-i$

which is not a real

$$\boxed{\mathbb{C} = \{a+bi \mid a,b \in \mathbb{R}\}}$$

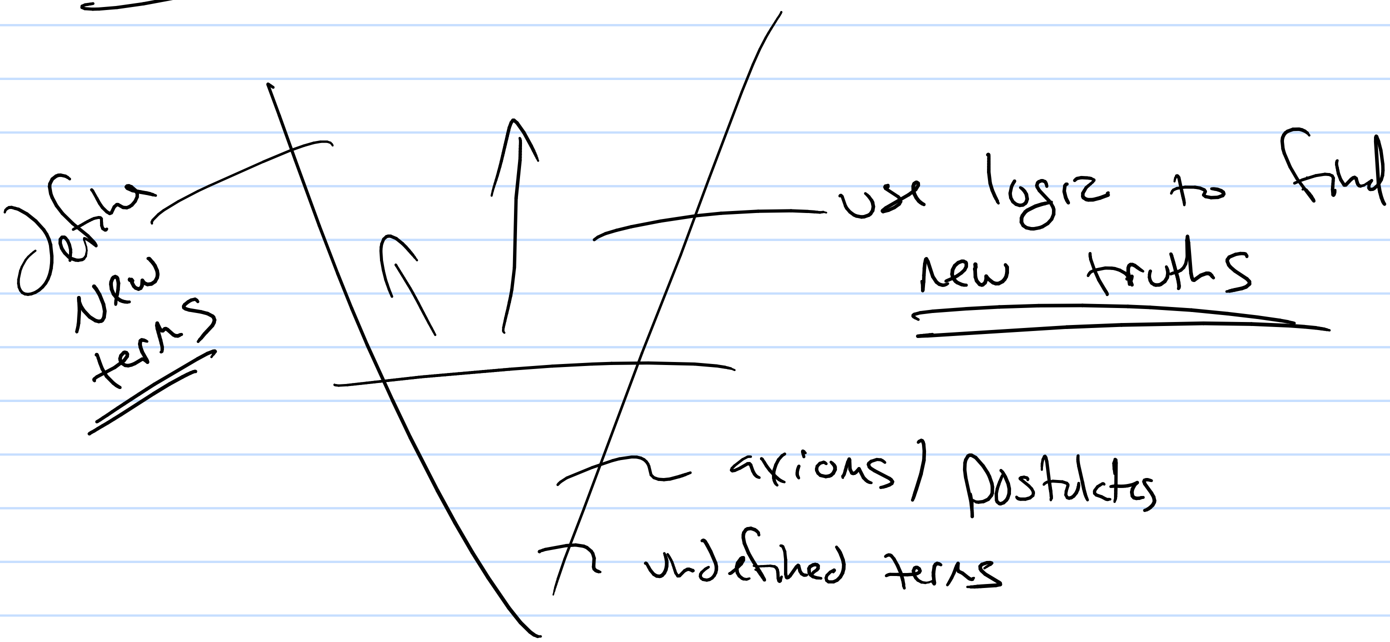
$\ni$: such that

$\in$ element of

Building Math(s)

Proofs 6

Back to Axiomatic theory



process of showing a statement is true is
the **proof** ← valid mathematical argument.

Names for statements

① unknown if true?

conjecture

② **prove it to be true**

fact , result.

if used to prove something important : lemma

important : theorem

really really important :

give it a name

back to less important is a natural
result of a theorem : **corollary**

less important

very important

$x^2 + y^2 = z^2$

$3^2 + 4^2 = 5^2$

Integer Solutions

$x + y = z$ ✓

$x^2 + y^2 = z^2$ ✓

$x^3 + y^3 = z^3$

$x^4 + y^4 = z^4$

$\boxed{x^p + y^p = z^p}$

Proof

conjecture ≡ statement we will try to prove.

Conjecture type 1

if p, then q

ex: if $\underbrace{\boxed{n \text{ is an even number}}}_{\text{hypothesis}}$, then $\underbrace{\boxed{n^2 \text{ is even}}}_{\text{conclusion}}$

warning is

p	q	p→q
T	T	T
T	F	F
F	T	T
F	F	T

← Vacuous

Direct Proof: assume hyp. is true and then show conclusion is also true.

Conjecture: if n is even, then n^2 is even.

or "the square of an even number is even"

Pf We will assume we have an even number, and call it n.

?. how do I represent all infinite even numbers?.

Scratch $2n$ where n is $-\infty, \infty$

even is $2 \cdot$ any integer

$n = 2\,(\text{int})$

n is even

$n = 2k$ and $k \in \mathbb{Z}$

$n \cdot n = n^2$

$n^2 = (2k)(2k)$

$= 2\,(2k^2)$

int

$n^2 = 2\,(\text{int})$

n^2 is even