Math 415

Monday -- No School!

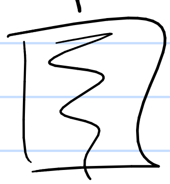
Today: Continue w/ Intro to Proofs

Wed: Review for Exam 1

- All HW assignments
- Examples from the text sections (Math Reasoning)
- Lecture Material

$\boxed{\text{HW Due Thurs}}$

① Show "Superman" argument is $\boxed{\text{Valid}}$ (true) if premises are true. (use verbal approach)



∴ Superman does not exist.

premises $\Longrightarrow$ conclusion

② take "Superman" argument as only variables. show it is valid (and therefor an argument form)

③ Verify several rules of inference by truth tables.

$$\begin{array}{c} p \to q \\ p \\ \hline \therefore q \end{array} \longrightarrow \boxed{[(p \to q) \wedge p] \to q}$$

Proofs: Read ch 4 of Book of Proofs.

Direct Proof:

for a typical "if hyp, then conclusion" conjecture.

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

find hyp is false

this is a vacuous truth

hyp conclusion

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

if conclusion is just true, the statement is true.

Trivial Proof.

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Direct Proof:

1. Assume the hyp.
2. use the tools and abilities you have to show conclusion must be true.

Need to know the "words" of the problem.

ex $\boxed{3 + 2}$

addition of Integers

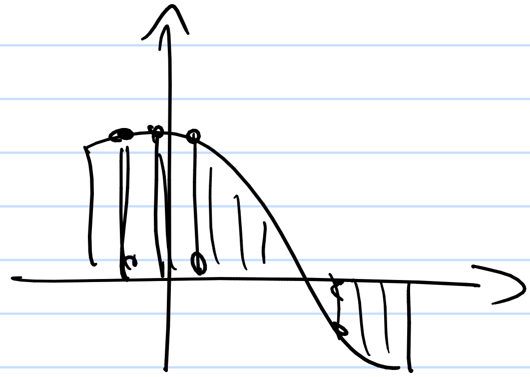
Integers

Operator Identity

Inverse

$$\int_a^b f(x)\,dx$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x_i$$



$$\int f(x)\,dx = F(x) + C$$

$$\frac{d}{dx}\left[F(x) + C\right] = f(x)$$

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Def: ① n is an even integer if

$n = 2 \cdot k$ for k an integer.

$\Updownarrow$

2 is a factor of integer n

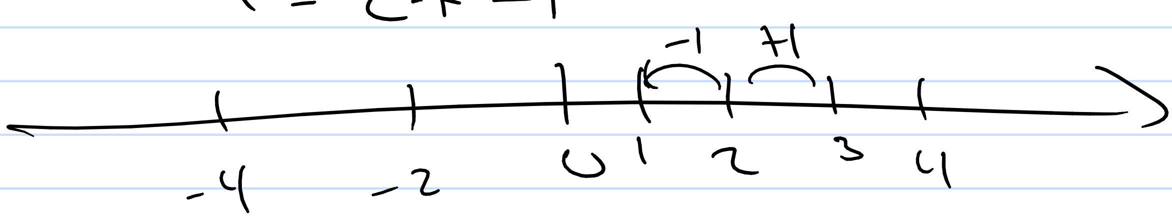
n is an integer multiple of 2.

② when n is not even we call it odd.

$n = 2 \cdot k + 1$ for k an integer

or

$n = 2 \cdot k - 1$



(3) parity of a number is if it's even or odd

ex what is 7's parity? odd

are $\underbrace{-5 \text{ and } 4}_{\text{are opposite parity}}$ of same parity? No

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Prove: addition of same parity is even.

"if $\underbrace{a, b \text{ are of same parity}}_{\text{hyp:}}$, then $\underbrace{a+b \text{ is even}}_{\text{conclusion.}}$"

[Pf] Direct proof

assume hyp and show conclusion is true.

assume a and b are integers. For a, b to be of same parity we have two cases. One, they are both even. Two, they are both odd.

Case One: a, b are even. By definition of even integers $a = 2k_1$, k_1 is an integer and
$b = 2k_2$, k_2 is an integer.
then $a+b = 2k_1 + 2k_2 = 2(k_1+k_2)$.

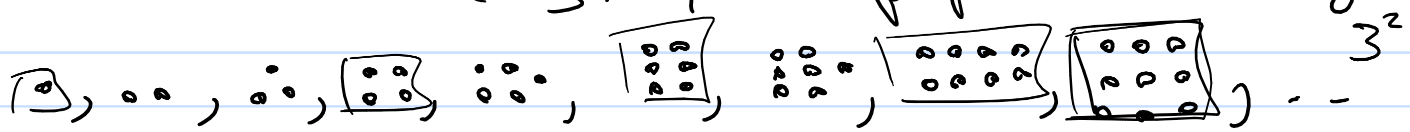
Because k_1 and k_2 are integers k_1+k_2 is an integer.
therefore $a+b = 2c$ where $c = k_1+k_2$ is an integer

and $a+b$ is even.

Case two: a, b are odd (see above for similar argument)

Def: (Number theory)

$\nwarrow$ Study the properties of integers



1, 2, 3, 4, 5, 6, 7, 8, 9, ...

divides a divides b if $\exists c \in \mathbb{Z}$ such that

$$a \cdot c = b$$

Symbol of

$|$

$a | b$ means $ac = b$ for some $c \in \mathbb{Z}$

$$S_{12} = \{ n \mid n | 12 \} = \{1, 2, 3, 4, 6, 12\}$$

such that divides

$$S_2 = \{ n \mid n | 2 \} = \{1, 2\}$$
$$S_3 = \{ n \mid n | 3 \} = \{1, 3\}$$
$$S_4 = \{ n \mid n | 4 \} = \{1, 2, 4\}$$
$$S_5 = \{ n \mid n | 5 \} = \{1, 5\}$$

Prove: if $\boxed{a \mid b \text{ and } a \mid c}$, then $\boxed{a \mid (b+c).}$

$\underbrace{\phantom{a \mid b \text{ and } a \mid c}}_{\text{hyp}}$ $\underbrace{}_{\text{conclusion}}$

proof: (direct?) assume $a \mid b \wedge a \mid c$. So $\boxed{a \cdot n = b}$ for n an integer and $\boxed{a \cdot m = c}$ for m an integer.

do math stuff

$\boxed{a \cdot d = (b+c)}$ for an integer d.

$\boxed{a \mid (b+c)}$