Math 415

Monday

Exam 1 12 probs @ 10 pts each

110 pts = 100%

ch 2 Mathematical Reasoning + Lectures

ch 2 Book of Proof (helpful)

① Statements .. Is it a proposition?

② Truth Table everyone should know

p q	$\neg p$	$\neg q$	$p \wedge q$	$p \vee q$	$p \oplus q$	$p \to q$	$p \leftrightarrow q$

w/ Q's about it

③
④
⑤ Make truth tables

(ex) $(p \wedge q) \to (r \vee \neg p)$

p	q	r	$\neg p$	$(p \wedge q)$	$(r \vee \neg p)$	$(p \wedge q) \to (r \vee \neg p)$
T	T	T	F	T	T	T
T	T	F	F	T	F	F
T	F	T	F	F	T	T
T	F	F	F	F	F	T
F	T	T	T	F	T	T
F	T	F	T	F	T	T
F	F	T	T	F	T	T
F	F	F	T	F	T	T

⑥ given English → translate into symbols (no quantification)

ex: If the sun is out and the day is cooler than 90°, then I will go for a run.

sun : "The sun is out"
day : "The day is cooler than 90° F"
run : "I will go for a run"

$(\text{sun} \wedge \text{day}) \to \text{run}$

w/ truth table

Q's under what conditions is this F?
T?

⑦ Symbols (w/ Quantification) → english sentences
Q's under what conditions is it F?
T?

ex:

$\forall x \in \mathbb{R} \left(\exists y \in \mathbb{R} \quad x \cdot y = 1 \right)$

"every real has a mult. inv." F

counter example $x = 0$ $\quad 0 \cdot (\text{nothing!}) = 1$

(8) given logical equiv. (ex: $\neg(p \wedge q) \equiv \neg p \vee \neg q$)

$p \to q \equiv \neg p \vee q$

Verify by truth tables

$\neg(p \to q) \equiv p \wedge \neg q$

show left $\equiv$ right

(A)

means: left $\Leftrightarrow$ right is always true.

(A)

p	q	$p \to q$	$\neg(p \to q)$	$\neg q$	$p \wedge \neg q$	$\neg(p\to q) \Leftrightarrow (p \wedge \neg q)$
T	T	T	F	F	F	T
T	F	F	T	T	T	T
F	T	T	F	F	F	T
F	F	T	F	T	F	T

Same

(9) Use logical Equiv. ("Simplify")

$$(p \to q) \vee (q \to p) \equiv (\neg p \vee q) \vee (\neg q \vee p)$$

$$\equiv \neg p \vee q \vee \neg q \vee p$$

$$\equiv (\neg p \vee p) \vee (\neg q \vee q)$$

$$\equiv T \vee T \equiv T$$

(10) for Rules of Inference

$$\begin{array}{l} p_1 \\ p_2 \\ p_3 \\ \vdots \\ p_n \\ \hline \therefore c \end{array}$$

Verify they are rules of inference.

by (1) truth table

(2) using logical equiv.

ex $\quad \dfrac{\begin{array}{c} p \to q \\ p \end{array}}{\therefore q} \qquad [(p \to q) \land p] \to q$

(1) logical equiv: $\quad [(p \to q) \land p] \Rightarrow q$

$$\equiv \neg[(p \to q) \land p] \lor q$$

$$\equiv [\neg(p \to q) \lor \neg p] \lor q$$

$$\equiv \neg(p \to q) \lor \underline{(\neg p \lor q)}$$

$$\equiv \neg(p \to q) \lor (p \to q) \equiv T$$

(2) truth table

(11) given multiple premises ... come up with valid conclusions.

ex
if Mark is happy, then Mark claps his hands.
Mark did not clap his hands.
Mark eats cheese.
if Mark eats cheese, then Mark is sad.

$\therefore$? $\quad$ c1: "Mark is not happy"

(12) given an argument .. Is it valid?

if valid? Verify!

if not valid? State why.

If Mark is happy, then he claps his hands.
Mark clapped his hands.

$\therefore$ Mark is happy.

Invalid. This argument is affirming the conclusion.

for example, Mark's hands could be covered in dirt and he is clapping to get the dirt off.

Direct Proofs:

conjecture type #1 : $(p \to q)$

(variation of this) : q

technique of a direct proof:

assume hyp. is true.

-3

10 $\boxed{1}$, 4, 2, 1, 4, 2, 1, 4, 2, 1

32

5

16

3, 10, 5, 16, 8, 4, 2, 1

8

4

2

1

$1, \boxed{2}, \boxed{3}, 2^2, \boxed{5}, 2\cdot 3, \boxed{7}, 2^3, 3^2, 2\cdot 5, \boxed{11}$

Conjecture: there are an infinite number of primes.

$1 \quad \boxed{2} \quad \boxed{3} \quad \boxed{5} \quad \boxed{7}$

$\boxed{11} \quad \boxed{13} \quad \boxed{17} \quad \boxed{19}$

$23 \quad 29$

...

Ex $a|b \wedge a|c \rightarrow a \mid b+c$

a, b, c are integers

$a|b$ means $\exists d \in \mathbb{Z}$ such that $a \cdot d = b$

Scratch

pf (direct)

start

assume $a|b$ and $a|c$.

means $\exists d \in \mathbb{Z}$ such that $a \cdot d = b$
and $\exists e \in \mathbb{Z}$ such that $a \cdot e = c$

$b+c = a\cdot d + a \cdot e$
$= a(d+e)$ int's

$a \cdot (\text{int}) = b+c \leftarrow$

$a | b+c$ | end

Pf We will use a direct proof. Therefore assume that $a|b$ and $a|c$. From this we know by def. of divides that there is an integer k_1 such that $a \cdot k_1 = b$ and an integer k_2 such that $a \cdot k_2 = c$.
Consider $b+c$, $b+c = a \cdot k_1 + a \cdot k_2$
$= a(k_1 + k_2)$

etc