Math 415

Book of Proof:

Ch4 Direct Proofs

Due Next Wed ch4(1,3,5,6,8,10,14,26)

Direct assume hyp and show conclusion.

ex products of two integers.

even or odd

play: $2\cdot3=6$
$3\cdot3=9$
$2\cdot4=8$
$5\cdot4=20$
⋮

$2\cdot3=6$
even odd even

$5\cdot4=20$
odd even even

$6\cdot8=48$
even even even

$5\cdot7=35$
odd odd odd

Conjecture #1 the product of two odd is odd

Conjecture If n is an odd integer and m is an odd integer, then $m\cdot n$ is an odd integer.

proof Using a direct proof we assume that m and n are odd. Therefore

Conjecture If n is an odd integer and m is an odd integer, then $m \cdot n$ is an odd integer.

proof Using a direct proof we assume that m and n are odd. Therefore, $m = 2 \cdot k_1 + 1$ where k_1 is an int. and $n = 2k_2 + 1$ where k_2 is an integer.

do "math stuff"

So $m \cdot n = (2k_1+1)(2k_2+1)$

$mn = 4k_1k_2 + 2k_1 + 2k_2 + 1$

$mn = 2(2k_1k_2 + k_1 + k_2) + 1$

By closure property of product and sum of integers $2k_1k_2 + k_1 + k_2$ is an integer, call it k_3

$mn = 2k_3 + 1$, k_3 is an integer

mn is odd

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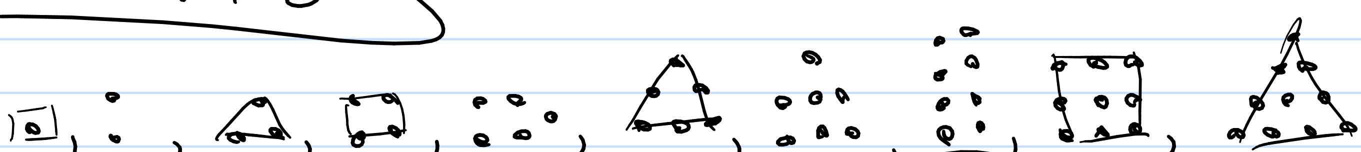
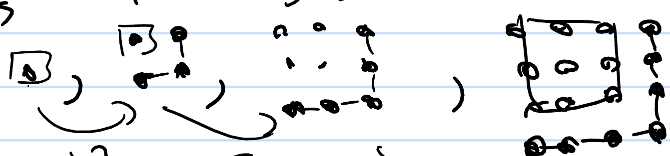
ex of playing



Squares

2×2 $\quad$ 3×3 $\quad$ 4×4



0, 1, ..., ...

$+3 = 2\cdot1+1$ $\quad$ $+5 = 2\cdot2+1$ $\quad$ $+7 = 2\cdot3+1$

Scratch

k^2 next $(k+1)^2$

$(k+1)^2 - k^2$

$= (k^2+2k+1) - k^2$

$= 2k+1$

Conjecture: every odd is the difference of two squares.

pf: assume n is an odd, therefore $n = 2k+1$, $k \in \mathbb{Z}$

So $n = 2k+1 = 2k+1+k^2-k^2 = k^2+2k+1-k^2$

$= (k+1)^2 - k^2$ which is a diff. of squares ∎

$\boxed{\text{Ch 5}}$ (1st Non-Direct technique)

use $(p \Rightarrow q) \equiv (\neg q \Rightarrow \neg p)$

contrapositive of original implication

$\boxed{\text{ex}}$ n^2 is odd, then n is odd

$\boxed{\text{try direct}}$ assume $n^2 = 2k_1 + 1$, $k_1 \in \mathbb{Z}$

"Math" stuff (can't ?!!)

$n = 2k_2 + 1$, $k_2 \in \mathbb{Z}$

n is odd

Try contrapositive:

$(n^2 \text{ is odd} \rightarrow n \text{ is odd})$

$\equiv (\neg(n \text{ is odd}) \rightarrow \neg(n^2 \text{ is odd}))$

$\equiv n \text{ is even} \rightarrow n^2 \text{ is even}$

$\boxed{\text{pf}}$ assume $n = 2k_1$, $k_1 \in \mathbb{Z}$

so $n^2 = (2k_1)^2 = 4k_1^2 = 2(2k_1^2)$

let $k_2 = 2k_1^2$ which is an integer (why?)

so $n^2 = 2k_2$ $\therefore$ n^2 is even.

by contrapositive we proved original.

$\blacksquare$

Conjecture: $x, y \in \mathbb{R}$

if $y^3 + yx^2 \le x^3 + xy^2$, then $y \le x$.

pf: try contraposition, so we will show

if $y > x$, then $y^3 + yx^2 > x^3 + xy^2$.

Assume $y > x$

we can multiply by non-zero, pos numbers and inequality does not change direction. Consider $x^2 + y^2$

$$y(x^2+y^2) > x(x^2+y^2)$$

so $y^3 + yx^2 > x^3 + xy^2$

Scratch

$y^3 + yx^2 > x^3 + xy^2$

$y(y^2+x^2) > x(x^2+y^2)$

$l > r$

direct: $y^3 + yx^2 \le x^3 + xy^2 \Rightarrow y \le x$

assume $y^3 + yx^2 \le x^3 + xy^2$

Same as $\dfrac{y(\cancel{y^2+x^2})}{(\cancel{x^2+y^2})} \le \dfrac{x(\cancel{x^2+y^2})}{(\cancel{x^2+y^2})}$

Case 1

$y \le x$ if $x = y = 0$ is not true.

Case 2 $x=0, y=0$ $0 \le 0 \rightarrow 0 \le 0$

ex $\quad x^2(y+3)$ is even $\rightarrow$ (x is even or y is odd)

try contrapositive:

if x is odd and y is even $\rightarrow$ $x^2(y+3)$ is odd

pf $\quad$ assume $x = 2k_1 + 1$, $k_1 \in \mathbb{Z}$ and $y = 2k_2$, $k_2 \in \mathbb{Z}$

$$\text{So } x^2(y+3) = (\overbrace{\boxed{2k_1+1}}^{x})^2(\overbrace{\boxed{2k_2}}^{y}+3)$$

$$= (4k_1^2 + 4k_1 + 1)(2k_2+3)$$

$$= \underline{8k_1^2k_2} + \underline{8k_1k_2} + \underline{2k_2} + \underline{12k_1^2} + \underline{12k_1} + 3$$

$2+1$

$$= 2(\underbrace{\text{I'm lazy!}}_{k_3 \text{ an integer}}) + 1$$

goal? $\quad x^2(y+3) = 2(k_3) + \underline{\underline{1}}$

is odd