Math 415

Due Tues ch5(2, 5, 6, 8, 9, 17, 18)

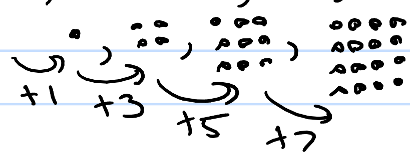
ch6(3, 4, 5, 8, 9)

Q's

26. Every odd integer is a difference of two squares. (Example $7 = 4^2 - 3^2$, etc.)

Scratch!

Squares: $0, 1, 4, 9, 16, \ldots$

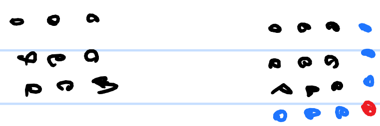


Hint: $7 = 4^2 - 3^2$

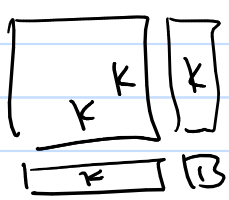
$3^2 + 7 = 4^2$

$3^2 + (2\cdot 3 + 1) = (3+1)^2$

3^2



$3^2 + \underbrace{(2\cdot 3 + 1)}_{\text{odd}} = 4^2$



$k^2 + (2k+1) = (k+1)^2$

Consider: $(k+1)^2 - k^2 = k^2 + 2k + 1 - k^2$

$= 2k+1$ $k = 0, 1, 2, 3, \ldots$

(26) If n is odd, then n is a subtraction of two squares.

Pf: assume n is odd. So $n = 2k+1$, $k \in \mathbb{Z}$

now

$2k+1 = 2k+1 + k^2 - k^2 = (k^2 + 2k + 1) - k^2$

$= (k+1)^2 - k^2$

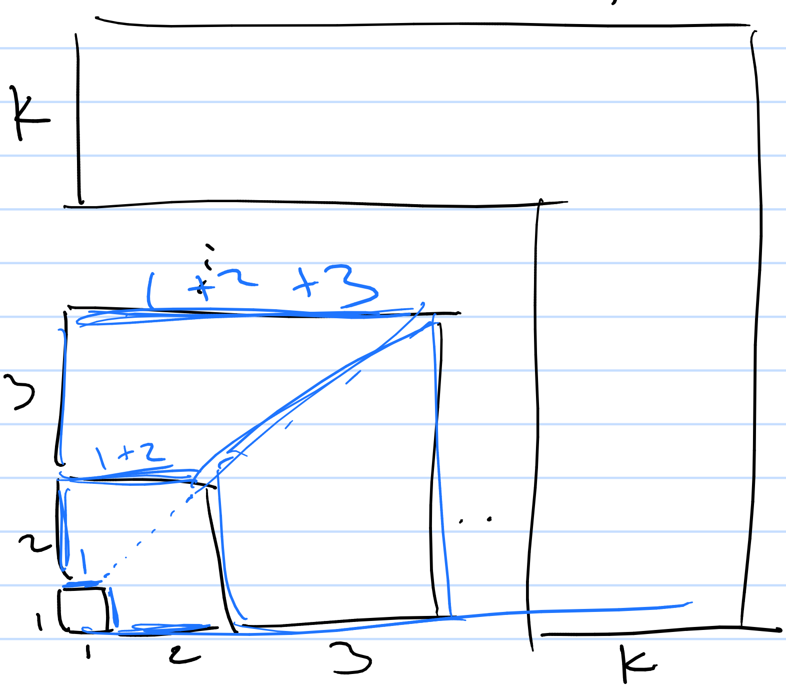
∎

Statement: every odd is a diff. of squares

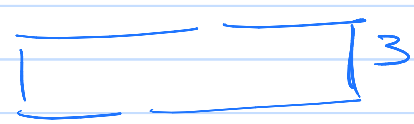
pf: $2k+1 = 2k+1+k^2-k^2 = (k^2+2k+1)-k^2$
$= (k+1)^2 - k^2$

Statement every square is the sum of consecutive triangular numbers.

Statement: the sum of the first k cubic numbers is the square of the k^{th} triangular number.



proof!



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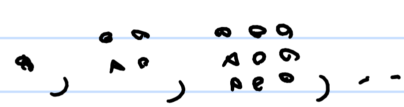
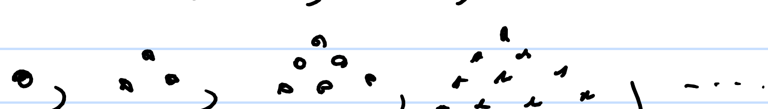
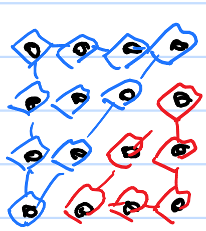
terms: squares: 

triangulars 

$k^{th}\ \Delta$ $1+2+3+\dots+k = \frac{k(k+1)}{2}$

$k + (k-1) + \dots \qquad + 1$

$(k+1) + (k+1) \qquad (k+1) = k(k+1)$

Note: 

by contradiction:

goal : "statement" $\equiv T$

rather do : $\neg$ "statement" $\equiv F$

$\boxed{\text{Statement}}$ there are infinite primes.

$\boxed{\text{pf}}$ by contradiction. Assume there are finite primes.

Means the set of primes are $\{p_1, p_2, p_3, p_4, \ldots, p_k\}$

where k is a finite number.

Fact: all numbers are prime or composite.

its prime factor is itself

has some prime factor

Consider $N = p_1 \cdot p_2 \cdot p_3 \cdot \ldots \cdot p_k + 1$

all numbers have at least one prime factor, so let's call N's prime factor to be p_*.

p_* is a factor of N. And by observation p_* is a factor of $\boxed{p_1 \cdot p_2 \cdot \ldots \cdot p_k}$.

Now consider $N - p_1 \cdot p_2 \cdot \ldots \cdot p_k = 1$

we can factor out p_* and $\underline{p_*}\left(\underbrace{\frac{N}{p_*}}_{\text{int}} - \underbrace{\frac{p_1 p_2 \cdots p_k}{p_*}}_{\text{int}}\right) = 1$

so p_* is a factor of 1. $\equiv F$

Non-Implication   Ch 7 (Book of Proof)

Used Direct / Contraposition / Contradiction on

$p \to q$ $\neg q \to \neg p$ $(p \wedge \neg q) \equiv F$

① if and only if $(p \leftrightarrow q)$

(ex) n is even if and only if $7n+4$ is even

$\boxed{\text{Th}^n}$ $p \leftrightarrow q$ | is $p \equiv q$

it is a tautology

$\boxed{\text{2 proof techniques}}$

$\boxed{1^{st}}$ Use a seq. of logical equiv.

$p \equiv \text{step 1} \equiv \text{step 2} \equiv \dots \equiv q$

College Algebra

Solve $x^2 - x - 6 = 0$

$\to (x-3)(x+2) = 0$

$x-3=0$ $x+2=0$

$\boxed{x=3 \quad x=-2}$

$\boxed{2^{nd}}$ $(p \leftrightarrow q) \equiv \boxed{(p \to q) \wedge (q \to p)}$

$\boxed{\text{case 1}}$ show $p \to q$

$\boxed{\text{case 2}}$ show $q \to p$

ex n is even $\boxed{\text{if and only if}}$ $7n+4$ is even

Pf We will prove the following two statements:

① if n is even, then $7n+4$ is even.

(Idea: use direct)

② if $7n+4$ is even, then n is even.

(Idea: use contraposition and rather prove "If n is odd, then $7n+4$ is odd)

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② Multiple logical equiv.

$$(P_1 \Leftrightarrow P_2 \Leftrightarrow P_3 \Leftrightarrow \cdots \Leftrightarrow P_k)$$

typically it is stated. $P_1 \equiv P_2 \equiv P_3 \equiv \cdots \equiv P_k$

Statement The following are logically equivalent

① P_1
② P_2
③ P_3

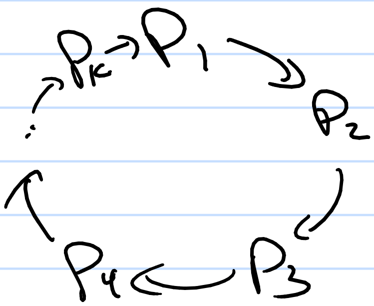
ex Math 511 The following are logically equiv.

① A is row equiv. to I
② $A\vec{x}=\vec{0}$ has only trivial solution
③ A^{-1} exists
④ A is non-singular
⑤ $\det(A) \neq 0$

to prove $p_1 \equiv p_2 \equiv p_3 \equiv \cdots \equiv p_k$

same as $(p_1 \to p_2) \land (p_2 \to p_3) \land \cdots \land (p_{k-1} \to p_k) \land (p_k \to p_1)$

<u>cycle</u>



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(3) <u>Cases:</u> $(p_1 \lor p_2 \lor p_3 \lor \cdots \lor p_k) \to q$

$\equiv \underset{\text{case 1}}{(p_1 \to q)} \land \underset{\text{case 2}}{(p_2 \to q)} \land \cdots \land \underset{\text{case } k}{(p_k \to q)}$

<u>Proofs by Cases</u>

if the cases are <u>finite</u> and you do prove all cases ..

<u>Proof by Exhaustion</u>

ex: if $(n+1)^3 < 3^n \to n > 4$

for n a positive integer.

pf: Try contraposition and prove if $n \le 4$, then $(n+1)^3 \ge 3^n$

$(n=1 \text{ or } n=2 \text{ or } n=3 \text{ or } n=4) \to (n+1)^3 \ge 3^n$

$(n=1 \text{ or } n=2 \text{ or } n=3 \text{ or } n=4) \Rightarrow (n+1)^3 \geq 3^n$

$\boxed{\text{case 1}}$ if $n=1$, then $(n+1)^3 \geq 3^n$

assume $n=1$, then $(n+1)^3$ vs 3^n is

$(2)^3 \geq 3^1$

<u>true?</u> yep.

$\boxed{\text{case 2}}$ if $n=2$, then $(n+1)^3 \geq 3^n$