Math 415

$\boxed{Q's}$

6. Suppose $x, y \in \mathbb{R}$. Then $x^3 + x^2y = y^2 + xy$ $\boxed{\text{if and only if}}$ $y = x^2$ or $y = -x$.

<u>conjecture.</u> $(x^3 + x^2y = y^2 + xy) \longleftrightarrow (y = x^2 \text{ or } y = -x)$

vs conjecture $(x^2 = 4)$ iff $(x = 2 \text{ or } x = -2)$

$\boxed{pf}$

$(x^2 = 4)$

$\equiv (x^2 - 4) = 0$

$\equiv (x+2)(x-2) = 0$

$\equiv x+2 = 0 \text{ or } x - 2 = 0$

$\equiv x = -2 \text{ or } x = 2$

vs $\boxed{\text{Case 1}}$ $(x^2 = 4) \Rightarrow (x = 2 \text{ or } x = -2)$

and

$\boxed{\text{Case 2}}$ $(x = 2 \text{ or } x = -2) \Rightarrow (x^2 = 4)$

<u>Conjecture</u> $(x^3 + x^2y = y^2 + xy) \longleftrightarrow (y = x^2 \text{ or } y = -x)$

<u>Scratch</u> (try start $\equiv$ step 1 $\equiv$ .. $\equiv$ end)

$(x^3 + x^2y = y^2 + xy)$

$\equiv (y^2 + xy - x^2y - x^3 = 0)$

$\equiv (y^2 + (x - x^2)y - x^3 = 0)$

$\equiv (y - x^2)(y + x) = 0$

$\equiv y - x^2 = 0 \text{ or } y + x = 0$

$\equiv (y = x^2 \text{ or } y = -x)$

$(x^3 + x^2y = y^2 + xy) \longleftrightarrow (y = x^2 \text{ or } y = -x)$

vs

Case 1

$(x^3 + x^2y = y^2 + xy) \rightarrow (y = x^2 \text{ or } y = -x)$

Case 2 $(y = x^2 \text{ or } y = -x) \rightarrow (x^3 + x^2y = y^2 + xy)$

Case 2.1 $y = x^2 \rightarrow (x^3 + x^2y = y^2 + xy)$

Case 2.2 $y = -x \rightarrow (x^3 + x^2y = y^2 + xy)$

Existence $\exists x\, P(x)$

→ actually find it: witness

constructive proof.

→ show $P(x)$ is true for some x in Univ. of Discourse (but do not exactly find the witness)

non-constructive proof

show: $A \oplus B \equiv T$

Uniqueness:

$\exists x (P(x) \wedge \boxed{\forall y} (P(y) \rightarrow (x = y)))$

Part 1 show $\exists x\, P(x)$

Part 2 show for $\boxed{P(y)} \rightarrow (x = y)$

$$\begin{bmatrix} a & \cdots \\ b & \cdots \end{bmatrix}$$

row esch. form?

make this a zero! by row ops

find r such that $a \cdot r + b = 0$

Conjecture: for reals, $a \neq 0$, $a \cdot r + b = 0$ has a uniq r.

Pf

Part 1 (existence)

let $r = -\frac{b}{a}$

$$a \cdot r + b = a\left(\frac{-b}{a}\right) + b = -b + b = 0$$

so $r = -\frac{b}{a}$ is a solution.

Part 2 (uniqueness)

assume some number, s, also is

$$a \cdot s + b = 0$$

$$as + b = ar + b \quad \text{b/c } ar + b = 0$$

$$as = ar$$

$$s = r$$

∎

---

other tech

Part 1 $\exists x\, P(x)$ (existence)

Part 2 $P(y) \to (x = y)$

$$\equiv (x \neq y) \to \neg P(y)$$

Another: part 2 $\big(P(y) \wedge (x \neq y)\big) \equiv F$

B) Exam 2 12 probs @ 10pts 110 pts = 100%

Next Wednesday

Proof types

Implication ($p \to q$)

1. direct for $p \to q$
2. contraposition for $p \to q$
3. contradiction for $p \to q$

cases of $p \to q$

4. $p \Leftrightarrow q$ as tech#1 $\boxed{p \equiv s_1 \equiv s_2 \equiv \dots \equiv s_k \equiv q}$

   as tech#2 (cases) case 1 $p \to q$ and case 2 $q \to p$

5. $p_1 \equiv p_2 \equiv p_3 \equiv \dots \equiv p_k$

   same as saying ...
   the following k-statements are equiv. ① p_1 ② p_2 ⋮ ⓚ p_k

   case 1 $p_1 \to p_2$
   case 2 $p_2 \to p_3$
   ⋮
   case k $p_k \to p_1$

6. $(p_1 \vee p_2 \vee \dots \vee p_k) \to q$

   case 1 $p_1 \to q$
   case 2 $p_2 \to q$
   ⋮
   case k $p_k \to q$

ex) $(y = x^2$ or $y = -x) \to ($ ~ $)$

from above

7. $\exists x P(x)$
8. $\exists x (P(x) \wedge \forall y (P(y) \to (x = y)))$

Direct Proof: (2 probs)

① direct like examples / homework / lecture

② ✱ variation on $\boxed{3}$

Involve divisible by 3 type prob.

Contraposition (2 probs)

① like examples / homework / lecture

② ✱ Needed lemma for $\boxed{3}$

Contradiction (2 probs)

① ✱ [prove $\sqrt{3}$ is irrational]

② Note: find another irrational.

biconditional proof $p \Leftrightarrow q$

1 prob

→ tech #1 $p \equiv s_1 \equiv s_2 \equiv \cdots \equiv q$

→ tech #2 $(p \to q) \wedge (q \to p)$

you pick the technique

logically equiv 0 prob

show
$p_1 \to p_2$
$p_2 \to p_3$
$p_3 \to p_1$

limit to 3 prop.

$p_1 \equiv p_2 \equiv p_3$

$a \equiv_m b$ Def: $m \mid (b-a)$

Th The following are log. equiv. (extra credit)

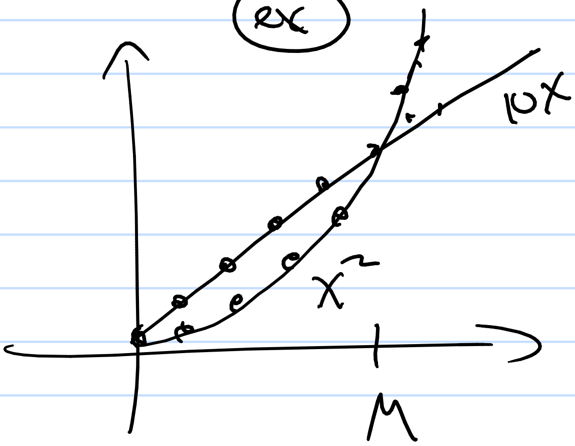
① $m \mid b-a$
② $b = a + km$
③ $a \bmod m = b \bmod m$

hand in on Wed.

2probs of $(p_1 \vee p_2 \vee \dots \vee p_k) \to q$

Note: contraposition could be used?

ex



$x \in \mathbb{Z}$

if $x^2 > 10x$, then $x > M$

$\equiv$ $x \le M \to x^2 \le 10x$

cases

Existance

3 probs

1. Constructive
2. Non-constructive
3. Uniqueness