Math 415

$\boxed{Q's}$

26. Concerning the Fibonacci sequence, prove that $\boxed{\sum_{k=1}^{n} F_k^2 = F_nF_{n+1}.}$

Same rule

$F_{k+5} = F_{k+4} + F_{k+3}$

$F_{k+1} = F_k + F_{k-1}$

$F_k = F_{k-1} + F_{k-2}$

Note: $\sum_{k=1}^{4} 3k+1 = (3\cdot1+1) + (3\cdot2+1) + (3\cdot3+1) + (3\cdot4+1)$

$$\sum_{k=1}^{n} F_k^2 = F_1^2 + F_2^2 + F_3^2 + \cdots + F_n^2$$

So $\sum_{k=1}^{n} F_n^2 = F_1^2 + F_2^2 + F_3^2 + F_4^2 + \cdots + F_n^2 = F_n F_{n+1}$

Remember: $F_0 = 0$ where $F_k = F_{k-1} + F_{k-2}$

$F_1 = 1$

$F_2 = 1$ $\boxed{\text{Basis}}$ 1st case: $F_1^2 = F_1 F_2$

$F_3 = 2$ $\quad 1 = 1\cdot1$ ✓ true!

$F_4 = 3$ $\quad \vdots$

$F_5 = 5$

$F_6 = 8$ $\quad k^{th}$: $F_1^2 + F_2^2 + \cdots + F_k^2 = F_k F_{k+1}$

$\vdots$

$k+1^{st}$: $F_1^2 + F_2^2 + \cdots + F_{k+1}^2 = F_{k+1} F_{k+2}$

Basis Step: show $F_1^2 = F_1 \cdot F_2$

well $1^2 = 1\cdot1$ is true!

Inductive Step: Assume the Inductive hypothesis.

show $F_1^2 + F_2^2 + \cdots + F_{k+1}^2 \stackrel{?}{=} F_{k+1} F_{k+2}$

We assume $F_1^2+F_2^2+\dots+F_k^2 = F_kF_{k+1}$

so $F_1^2+F_2^2+\dots+F_k^2+F_{k+1}^2 = F_kF_{k+1}+F_{k+1}^2$

$\qquad\qquad || \qquad\qquad = F_{k+1}(F_k+F_{k+1})$

b/c $F_k+F_{k+1} = F_{k+2}$ (*)

$F_1^2+F_2^2+\dots+F_{k+1}^2 \overset{*}{=} F_{k+1}F_{k+2}$

---

1   1

1 1   2

1 2 1   4

1 3 3 1   8

1 4 6 4 1   16

$\boxed{1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1}$

1 1 2 3 5

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

$(x+y)^5 = 1x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

$\frac{5!}{5!} = 1$

xxxyy

$10 = \frac{5!}{3!\,2!} = \binom{5}{2}$

$F_1 = 1$, $F_2 = 1$, $F_3 = 2$, $F_4 = 3$, ...

1, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...

$1^2 = 1 \cdot 1$

$1^2 + 1^2 = 1 \cdot 2$

$1^2 + 1^2 + 2^2 = 2 \cdot 3$

$\longrightarrow 1^2 + 1^2 + 2^2 + 3^2 = 3 \cdot 5$

$\longrightarrow 1^2 + 1^2 + 2^2 + 3^2 + 5^2 = 5 \cdot 8$

---

16. Prove that $2^n + 1 \le 3^n$ for every positive integer n. $= 1, 2, 3, 4, 5, 6, \ldots$

Cases

1^{st}: $(n=1)$ $\quad 2^1 + 1 \le 3^1$ Basis

2^{nd}: $(n=2)$ $\quad 2^2 + 1 \le 3^2$ Assume

3^{rd}: $(n=3)$ $\quad 2^3 + 1 \le 3^3$ I.H.

⋮

k^{th} $\quad 2^k + 1 \le 3^k$

$k+1^{st}$ $\quad 2^{k+1} + 1 \le 3^{k+1}$ show

Pf: Basis Step: show $2^1 + 1 \overset{?}{\le} 3^1$

$3 \le 3$ true!

Inductive Step: Assume I.H.

Scratch Paper:

Have: $2^k + 1 \le 3^k$ Want: $2^{k+1} + 1 \le 3^{k+1}$

$-1 \quad -1$

(*) $2^k \le \underbrace{3^k - 1}_{} \le \underbrace{3^k - 1}_{} + 1 = 3^k$

$2 \cdot 2^k + 1 \overset{?}{\le} 3 \cdot 3^k$

try #1 $2^{k+1} + 1 = 2 \cdot 2^k + 1 = 2^k + 2^k + 1$

$\le 2^k + 1 + 2^k + 1 \le 3^k + 3^k \le 3^k + 3^k + 3^k$

$= 3^{k+1}$

try #2

$2^k + 1 \le 3^k$

$2^k + 1 + 2^k \le 3^k + 2^k$

$2 \cdot 2^k + 1 \le 3^k + 2^k$

$2^{k+1} + 1 \le 3^k + 2^k \overset{(*)}{\le} 3^k + 3^k$

So $2^{k+1} + 1 \le \underbrace{3^k + 3^k}_{} \le \underbrace{3^k + 3^k}_{} + 3^k$

b/c 3^k is a pos. number.

So $2^{k+1} + 1 \le 3 \cdot 3^k = 3^{k+1}$

What Now? (Now that you all can prove anything!)

$\boxed{\text{Find Stuff to prove}}$

Sets and Proofs on Sets — Reason ch 5, BoP ch 8

Exam 3 on 11/1

Relations (sets of ordered pairs)

Functions (special relations) — Exam 4

Cardinality — BoP ch 11, 12, 14 — 11/15

Number theory and Proofs in Calculus — Exam 5

Reason ch 8 / BoP ch 13 — 12/1

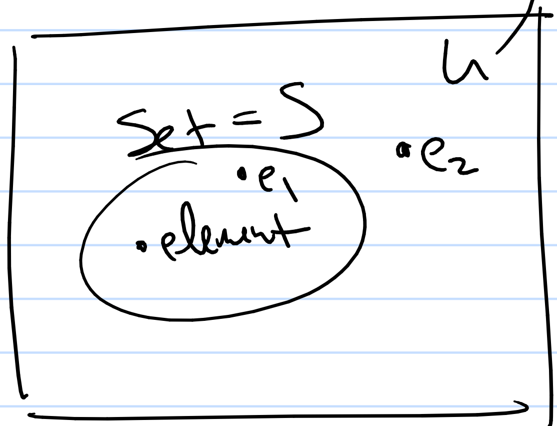
Set theory

unordered collection of elements.

Represent a Set

① Venn Diagram

Universe of Discourse



e_1 is in S, $e_1 \in S$

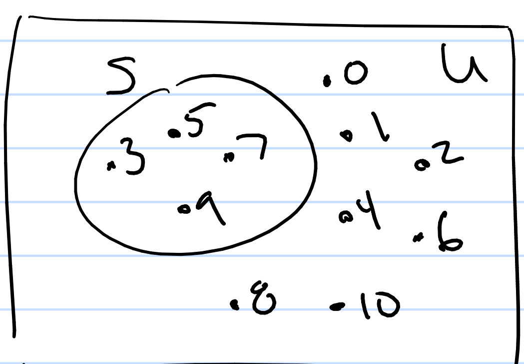
e_2 is not in S, $e_2 \notin S$

② Set Builder notation

$$S = \{ e_i \mid P(e_i) \}$$

variable element — such that — propositional function for a test to see when $e_i \in S$

ex) $S = \{ n \mid n \in \mathbb{Z} \wedge 2 \le n \le 10 \wedge n \text{ is odd} \}$

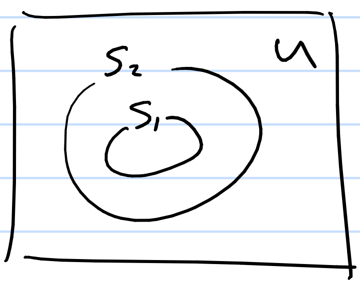


Universe of Discourse is all ints from 0 to 10

③ List or Roster $S = \{3, 5, 7, 9\}$

① Compare Sets?

(a) Subset $S_1 \subseteq S_2$



$\forall e \, (e \in S_1 \rightarrow e \in S_2)$

Special Sets ① $\phi = \{ \}$ empty set

② $U = \{$ all elements in U.D $\}$

Fact: ① $\phi \subseteq S$ for any set S.

$$\forall e \, (\underbrace{e \in \phi}_{F} \rightarrow e \in S)$$

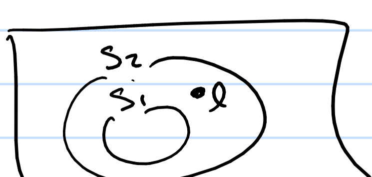
② $S \subseteq S$ for any set S.

$$\forall e \, (e \in S \rightarrow e \in S)$$

③ $S \subseteq U$

(b) Proper Subset $S_1 \subset S_2$

$$\forall e \, (e \in S_1 \rightarrow e \in S_2) \land \exists l \, (l \in S_2 \land l \notin S_1)$$



(c) $S_1 = S_2$

$$\forall e\,(e \in S_1 \leftrightarrow e \in S_2)$$

$$= \forall e\,((\underbrace{e \in S_1 \rightarrow e \in S_2}) \land (\underbrace{e \in S_2 \rightarrow e \in S_1}))$$

$$\equiv (S_1 \subseteq S_2) \land (S_2 \subseteq S_1)$$

(d) finite cardinality,

$|S|$ = count the unique elements

ex $|\{1, 1, 1, 2, 7\}| = 3$

Operations

1. Power Set
2. Cross Product
3. Union
4. Intersection
5. Difference
6. Complements