Math 415

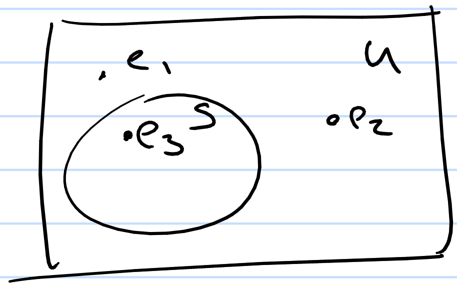
Due Monday BoP ch8 (1, 2, 7, 10, 11, 18, 21, 24)

Set Theory

Set ① $S = \{$ roster or list $\}$

② Set builder $S = \{ e \mid P(e) \}$ $\quad e \in S$ if $P(e) \equiv T$

③ Venn Diagrams



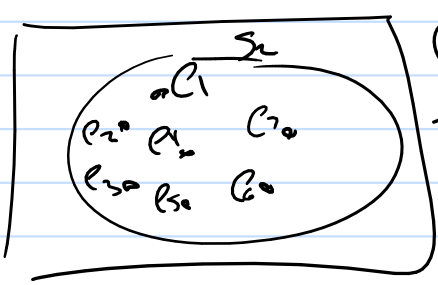
$e_3 \in S$

$e_1 \notin S$

Power Set

Power Set of a set $S = \{e_1, e_2, e_3, \dots e_k\}$

is the set that collects all subsets of S.



(ex) $P(S) = \{ \emptyset,$ Set w/ no elements. Need all Subsets

$\{e_1\}, \{e_2\}, \dots, \{e_7\}$ Set's w/ exactly 1 element

of 7 choose 2 $= \frac{7!}{2!5!}$ $\quad \{e_1, e_2\}, \{e_1, e_3\}, \dots, \{e_6 e_7\}$

$\vdots$

choose all $\{e_1, e_2, \dots, e_7\} = S \}$

(ex) $P(\{a,b,c\}) = \{ \boxed{\emptyset}, \boxed{\{a\}}, \boxed{\{b\}} \boxed{\{c\}}, \boxed{\{a,b\}}, \boxed{\{a,c\}}, \boxed{\{b,c\}}, \boxed{\{a,b,c\}} \}$

$|P(\{a,b,c\})| = 2^3 = 8$

Cardinality

$|P(S)|$ = number of **subsets** of S

→ order S = $e_1, e_2, e_3, \ldots, e_k$

How to make subsets?

$$\begin{array}{ccccc} e_1 & , & e_2 & \ldots & , e_k \\ Y ? N & & Y ? N & & Y ? N \\ 1 \text{ or } 0 & & & & \end{array}$$

S is $1, 1, 1, 1, \ldots, 1$ (K - 1's)

ϕ is $0, 0, 0, 0, \ldots, 0$

$\{e_1\}$ $1, 0, 0, 0, \ldots, 0$

1) so $|P(S)| = 2^{|S|}$

2) any subset of n elements of the $|S|$ total is $|S|$ choose n.

Note: **Membership table** $1 \equiv$ "in" $0 \equiv$ "out"

ex $S = \{a, b, c\}$

a	b	c	
1	1	1	$\{a,b,c\}$
1	1	0	$\{a,b\}$
1	0	1	$\{a,c\}$
1	0	0	$\{a\}$
0	1	1	
0	1	0	
0	0	1	
0	0	0	ϕ

Cross Product take sets $A_1, A_2, A_3, \ldots, A_n$

make a set of n-tuples where

$(a_1, a_2, \ldots, a_n)$ is all possible n-tuples

for $a_1 \in A_1$, $a_2 \in A_2, \ldots,$ $a_n \in A_n$

$$A_1 \times A_2 \times \cdots \times A_n = \{ (a_1, a_2, \ldots, a_n) : a_1 \in A_1 \land a_2 \in A_2 \land \cdots \land a_n \in A_n \}$$

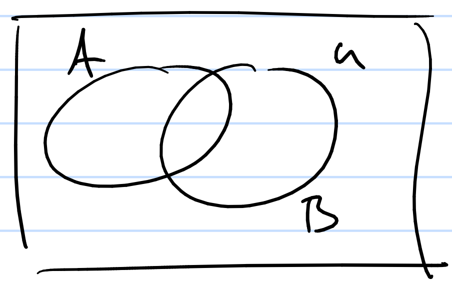
ex $A_1 = \{ \text{Mark}, \text{Joe} \}$

$A_2 = \{ \text{prime}, \text{composite}, \text{apple} \}$

$$A_2 \times A_1 = \{ \boxed{(\text{prime}, \text{Mark})}, \boxed{(\text{prime}, \text{Joe})}, \boxed{(\text{composite}, \text{Mark})}, \boxed{(\text{composite}, \text{Joe})}, \boxed{(\text{apple}, \text{Mark})}, \boxed{(\text{apple}, \text{joe})} \}$$

Note: $|A_1 \times A_2 \times \cdots \times A_n| = |A_1| \, |A_2| \cdots |A_n|$

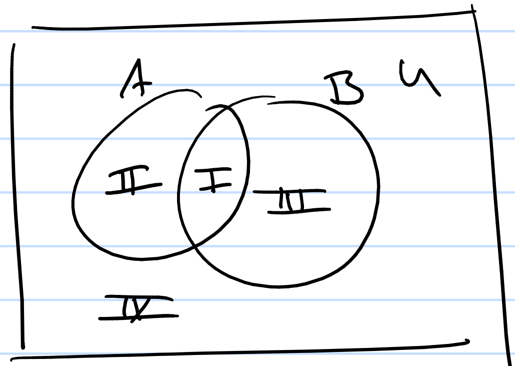
Operations in U.D.



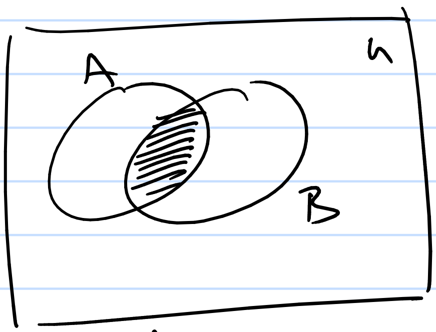
1. Union: $A \cup B = \{ e \mid e \in A \lor e \in B \}$
2. Intersection $A \cap B = \{ e \mid e \in A \land e \in B \}$
3. Difference $A - B = \{ e \mid e \in A \land e \notin B \}$
4. Complement $\bar{A} = U - A = \{ e \mid e \notin A \}$

Operations in V.D.

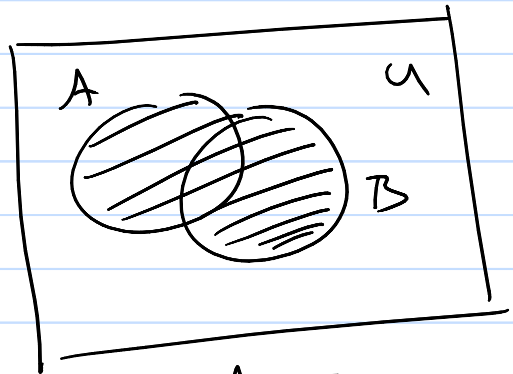
with membership tables



set/region	A	B	$A\cap B$	$A\cup B$
I	1	1	1	1
II	1	0	0	1
III	0	1	0	1
IV	0	0	0	0



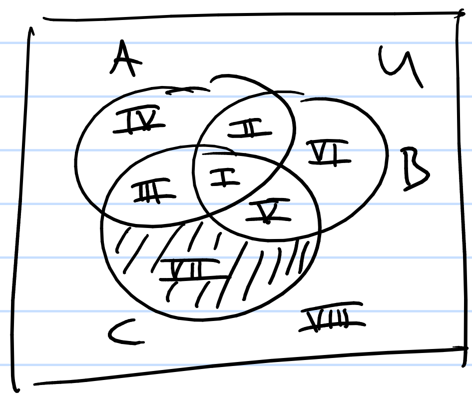
$A\cap B$



$A\cup B$

Visualize: $\overline{A\cup B}\cap C$

A	B	C	$A\cup B$	$\overline{A\cup B}$	$\overline{A\cup B}\cap C$
1	1	1	1	0	0
1	1	0	1	0	0
1	0	1	1	0	0
1	0	0	1	0	0
0	1	1	1	0	0
0	1	0	1	0	0
0	0	1	0	1	1
0	0	0	0	1	0



Identities

$$A\cap U = \{e \mid e\in A \wedge \underline{e\in U}\}$$
$$= \{e \mid \underline{e\in A \wedge T}\}$$
$$= \{e \mid e\in A\} = A$$

$$A\cap \phi = \{e \mid e\in A \wedge e\in \phi\}$$
$$= \{e \mid e\in A \wedge F\} = \{e \mid F\} = \phi$$

$A \cap U = A$ $\quad$ $A \cup \phi = A$ $\quad$ Identity laws

$A \cap \phi = \phi$ $\quad$ $A \cup U = U$ $\quad$ Domination laws

$A \cup A = A$ $\quad$ $A \cap A = A$ $\quad$ Idempotent laws

$\overline{\overline{A}} = A$ $\quad$ double complement law

(+) Commutative, Associative, Distributive, DeMorgan's

$\boxed{ex}$

$$\overline{A \cup B} = \{e \mid e \notin A \cup B\}$$
$$= \{e \mid \neg\,(e \in A \cup B)\}$$
$$= \{e \mid \neg\,(\underline{e \in A} \vee \underline{e \in B})\}$$
$$= \{e \mid \neg(e \in A) \wedge \neg(e \in B)\}$$
$$= \{e \mid e \in \overline{A} \wedge e \in \overline{B}\}$$
$$= \{e \mid e \in \overline{A} \cap \overline{B}\}$$

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

Proofs in Set theory

(ex) $A \subseteq B$ if and only if $P(A) \subseteq P(B)$

Biconditional Proof?

Sketch

try

case 1 $(A \subseteq B) \Rightarrow (P(A) \subseteq P(B))$

case 2 $(P(A) \subseteq P(B)) \Rightarrow (A \subseteq B)$

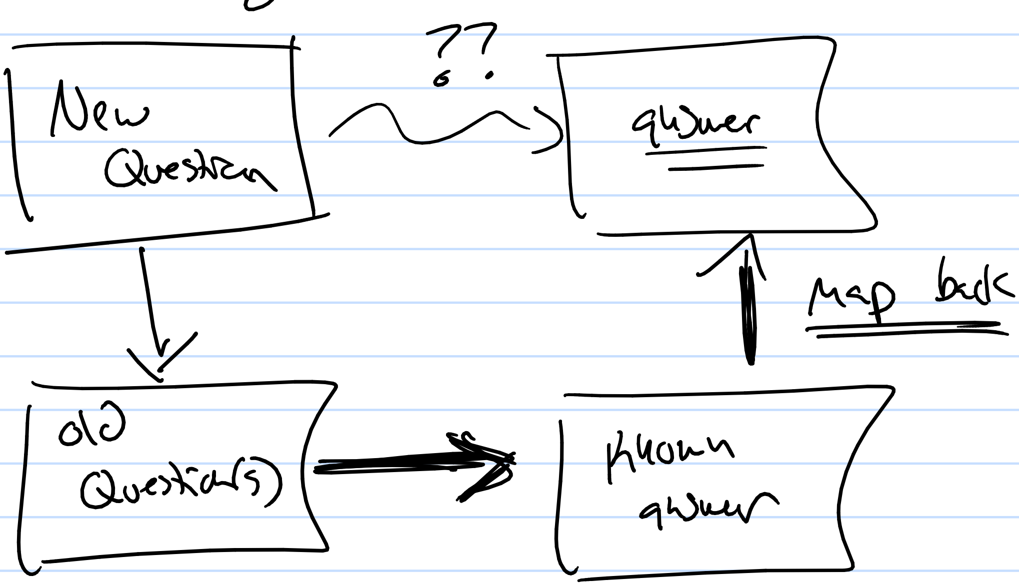
$A \subseteq B$

$P(A) = \{\emptyset, \{a\}, \{b\}, \{a,b\}\}$

$P(B) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$

See? $P(A) \subseteq P(B)$

Note: (Problem Solving)



$\boxed{\text{Case 1}}$ $A \subseteq B \rightarrow P(A) \subseteq P(B)$

Assume $\underline{A \subseteq B}$ $\boxed{\text{Goal!}}$ show $\boxed{P(A) \subseteq P(B)}$

Means: Means

(*) $e \in A \rightarrow e \in B$ set

is true. $S \in P(A) \rightarrow S \in P(B)$

means

$\boxed{S \subseteq A \rightarrow S \subseteq B}$

means

goal show true $\boxed{(e \in S \rightarrow e \in A) \rightarrow (e \in S \rightarrow e \in B)}$

Assume $e \in S \rightarrow e \in A$. But by 1st Assumption (*)

$e \in S \rightarrow e \in A \rightarrow e \in B$ so

$e \in S \rightarrow e \in B$ is true Case #1