Math 415

all numbers with a factor of 12 — A

all numbers of a factor of 6 $(2 \wedge 3)$ — B

Q's

1. Prove that $\{12n : n \in \mathbb{Z}\} \subseteq \{2n : n \in \mathbb{Z}\} \cap \{3n : n \in \mathbb{Z}\}$.

to show $A \subseteq B$

means $\forall e \, (e \in A \rightarrow e \in B)$

Scratch ?

$e \in \{12n \mid n \in \mathbb{Z}\}$

$e \in \boxed{\{\ldots, -24, -12, 0, 12, 24, \ldots\}}$ A

B is $\boxed{\{2n \mid n \in \mathbb{Z}\} \cap \{3n \mid n \in \mathbb{Z}\}}$

$\{\ldots, -6, -4, -2, 0, 2, 4, 6, \ldots\} \cap \{\ldots -6, -3, 0, 3, 6, \ldots\}$

$B = \{\ldots, -18, -12, -6, 0, 6, 12, 18, \ldots\}$

$B = \{e \mid e = 2n \wedge e = 3n\}$

$B = \{e \mid 2 | e \wedge 3 | e\}$

$B = \{e \mid \boxed{2 \cdot 3 \cdot n = e}\}$

e has factors of 2 and 3

Trex $\{x \mid x = 12n, n \in \mathbb{Z}\} \subseteq \{x \mid x = 2n, n \in \mathbb{Z}\} \cap \{x \mid x = 3n, n \in \mathbb{Z}\}$

Tpf for the set $\{x \mid x = 12n, n \in \mathbb{Z}\}$ it is all numbers with a factor of 12. We see that the intersection of $\{x \mid x = 2n, n \in \mathbb{Z}\}$ and $\{x \mid x = 3n, n \in \mathbb{Z}\}$ gives all numbers with a factor of 6.

Continue

Pf assume $12 \mid x$ so $x = 12 \cdot k$, $k \in \mathbb{Z}$

this gives $x = 2 \cdot 2 \cdot 3 \cdot k$, $k \in \mathbb{Z}$

we see $2 \mid x$ and $3 \mid x$.

therefore: $\{12n \mid n \in \mathbb{Z}\} \subseteq \{2n \mid n \in \mathbb{Z}\} \cap \{3n \mid n \in \mathbb{Z}\}$.

18. If A, B and C are sets, then $A \times (B - C) = (A \times B) - (A \times C)$.

show Set1 $\stackrel{?}{=}$ Set2

tech #1 $\boxed{\text{Membership Tables}}$ if all sets are in same Univ. of Disc.

$\overline{A \cup B} = \overline{A} \cap \overline{B}$

A	B	$A \cup B$	$\overline{A \cup B}$	$\overline{A}$	$\overline{B}$	$\overline{A} \cap \overline{B}$
1	1	1	0	0	0	0
1	0	1	0	0	1	0
0	1	1	0	1	0	0
0	0	0	1	1	1	1

by Venn Diagram these are the same regions

tech #2 Set Builder Notation.

Set1 = { e | Set 1's prop. function }

$\equiv$

= { e | logically same as above }

⋮

= { e | set 2's prop function } = Set 2

tech #3 Show Set1 $\subseteq$ Set2

and Set2 $\subseteq$ Set1

So

18. If A,B and C are sets, then $A \times (B - C) = (A \times B) - (A \times C)$.

$$A \times (B - C) = \{ (x,y) \mid x \in A \wedge y \in (B - C) \}$$

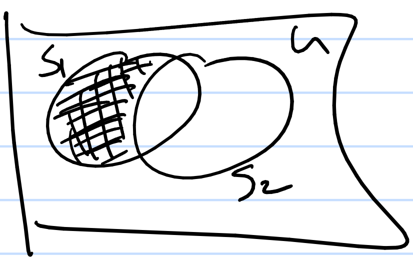
$$(A \times B) - (A \times C) = \{ (x,y) \mid (x,y) \in A \times B \wedge (x,y) \notin A \times C \}$$

ex of understanding cross product of two sets

$S_1 = \{ \square, \triangle \}$ $\quad S_2 = \{ \smile, \frown \}$

$$S_1 \times S_2 = \{ (\square, \smile), (\square, \frown), \boxed{(\triangle, \smile)}, (\triangle, \frown) \}$$

ex of understanding Difference



$$S_1 - S_2 = \{ e \mid e \in S_1 \wedge e \notin S_2 \}$$

$$A \times (B - C) = \{ (x,y) \mid x \in A \wedge y \in (B - C) \}$$

$$= \{ (x,y) \mid x \in A \wedge y \in B \wedge \neg (y \in C) \}$$

$$= \{ (x,y) \mid F \vee (x \in A \wedge y \in B \wedge \neg (y \in C)) \}$$

$$= \{ (x,y) \mid [x \in A \wedge y \in B \wedge \neg (x \in A)] \vee [x \in A \wedge y \in B \wedge \neg (y \in C)] \}$$

$$= \{ (x,y) \mid (x \in A \wedge y \in B) \wedge (\neg (x \in A) \vee \neg (y \in C)) \}$$

$$= \{ (x,y) \mid (x \in A \wedge y \in B) \wedge \neg (x \in A \wedge y \in C) \}$$

$$\{ (x,y) \mid (x,y) \in A \times B \wedge \neg ((x,y) \in A \times C) \} = (A \times B) - (A \times C)$$

$\boxed{\text{Exam 3}}$ Take Home given Monday Due Wed.

11 proofs

→ 6 are induction
→ 5 involve set theory.

1, 2] Induction using equalities

ex $1^3+2^3+3^3+\cdots+n^3 = \left(\frac{n(n+1)}{2}\right)^2$

k^{th} $\quad 1^3+2^3+\cdots+k^3 = \left(\frac{k(k+1)}{2}\right)^2 \leftarrow$ I.H.

$k+1^{st}$ $\quad 1^3+2^3+\cdots+(k+1)^3 = \boxed{1^3+2^3+\cdots+k^3} + (k+1)^3$

$$= \left(\frac{k(k+1)}{2}\right)^2 + (k+1)^3$$

$$= \frac{k^2(k+1)^2}{4} + (k+1)^3$$

$$= (k+1)^2\left[\frac{k^2}{4} + \frac{4(k+1)}{4}\right]$$

$=?$

$$= \left(\frac{(k+1)(k+2)}{2}\right)^2$$

(3) Inequality Inductive Proof

(4) Divisibility proof.

(5)
(6) equations using $\boxed{F_0 = 0 \quad F_1 = 1 \quad F_n = F_{n-1} + F_{n-2}}$

<u>Using Sets</u> (7) <u>Induction using Sets.</u>

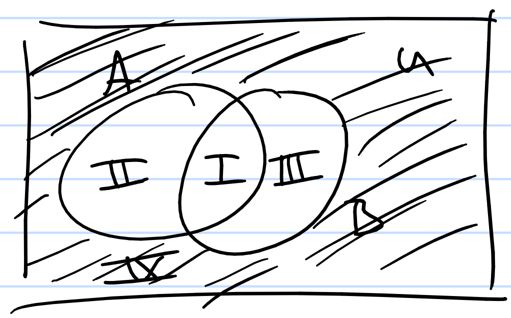
(ex) $\overline{S_1 \cap S_2 \cap \cdots \cap S_n} = \overline{S_1} \cup \overline{S_2} \cup \cdots \cup \overline{S_n}$

$n = 1, 2, \ldots$

(8) <u>Equality</u> by Membership Table (+) Venn Diagram

$\overline{A \cup B} = \overline{A} \cap \overline{B}$

A	B	$A \cup B$	$\overline{A \cup B}$
1	1	1	0
1	0	1	0
0	1	1	0
0	0	0	1



(9) Prove Subset

(10) Prove Equality

(11) <u>Prove:</u> $A \subseteq B$ iff $P(A) \subseteq P(B)$