Math 415

(3) if $n > 4$, $2^n > n^2$ $\quad n \in \mathbb{Z}$

Basis: $n = 5$, $2^5 > 5^2$ true

Inductive: assume $2^k > k^2$ $\quad$ (show $2^{k+1} > (k+1)^2$)

$2 \cdot 2^k$ $\quad k^2 + 2k + 1$

$2^k + 2^k$

$$2^{k+1} = 2 \cdot 2^k = 2^k + 2^k > k^2 + k^2$$

$$2^{k+1} > k^2 + \boxed{k^2} > k^2 + \boxed{2k+1}$$

$$k^2 > 2k + 1$$

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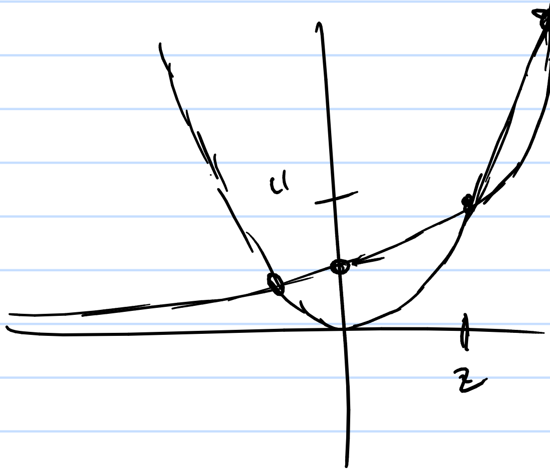
if $x > 4$, then $2^x > x^2$ $\quad x \in \mathbb{R}$

$$2^x - x^2 > 0$$

$$f(x) = 2^x - x^2$$



DoP

ch 11

Due Next Wed

11.1 (2, 4, 11)
11.2 (example 11.8 but do ref, irref, sym, antisym, asym, and trans
2, 8, 12) also do antisym.
11.3 (2, 7, 8)
11.4 (2, 3)

Equiv. Relations

R on set A is an equiv. relation if it is
① reflexive
② symmetric
③ transitive

ex for functions that are polynomials with real coeff.

Define a relation R such that $R = \{ (f, g) \mid f$ and g have same degree $\}$

ex $(4x^7 - 2x^3 + 1) \; R \; (\pi x^7 - 2x^2)$ a

Properties ① reflexive? $\forall f \, (f R f)$

all real coeff polynomials have the same degree as themselves. true.

② irreflexive? $\forall f \, (f \not R f)$ False counterexample $(x^2) R (x^2)$

③ symmetric? $\forall f_1 \forall f_2 \; (f_1 R f_2 \rightarrow f_2 R f_1)$ true.

④ antisym? $\forall f_1 \forall f_2 \; (f_1 R f_2 \land f_2 R f_1 \rightarrow f_2 = f_1)$
$\forall f_1 \forall f_2 \; (f_2 \neq f_1 \rightarrow \neg (f_1 R f_2 \land f_2 R f_1))$

⑤ asymmetric? $\forall f_1 \forall f_2 \, (f_1 R f_2 \rightarrow f_2 \not R f_1)$

(6) transitive $\forall f_1 \forall f_2 \forall f_3 (f_1 R f_2 \wedge f_2 R f_3 \to f_1 R f_3)$

true.

R is ref., sym, and trans. So R is an equiv. relation

Equiv. Classes

is a set that contains all elements related to a representative of the class for an Equiv. Relation

Notation: $[\text{representative}]_{R}$

leave off if "everyone" knows what relation you are talking about.

(ex) $\underbrace{[x^2+1]_R}_{\text{same degree polynomials}} = \{ p(x) \mid p \text{ is a polynomial w/ real coeff and } p R (x^2+1) \}$

$= \{ p(x) \mid p(x) = ax^2 + bx + c \wedge a,b,c \in \mathbb{R} \wedge a \neq 0 \}$

(ex) $R = \{ (a,b) \mid a \equiv_m b \}$ on the set $\mathbb{Z}$

ref? $\forall a (aRa)$

For all ints is $a \equiv_m a$?

$m \mid (a-a)$ is $m \mid 0$

True.

sym? $a \equiv_m b \to b \equiv_m a$

? For all ints a,b if $m \mid (b-a) \to m \mid (a-b)$?

if $mk = (b-a) \to (a-b) = -mk = m(-k)$

true

trans?: $\forall a,b,c\ (aRb \wedge bRc \to aRc)$

$$a \equiv_n b \wedge b \equiv_n c \to a \equiv_n c$$

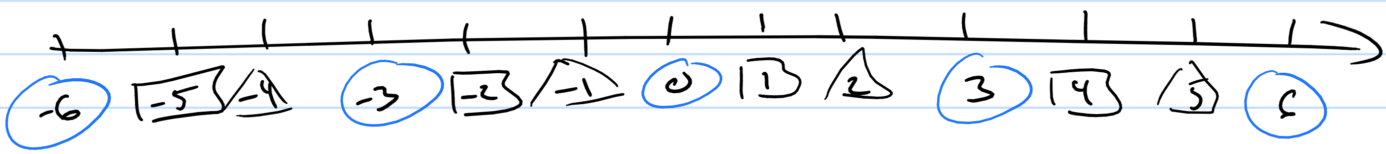
$$n \mid (b-a) \wedge n \mid (c-b)$$

so $n \mid (c-b)+(b-a)$

$n \mid (c-a)$ is $a \equiv_n c$

true.

Equiv Classes:



$[0]_{\text{div by } 3} = \{\ldots, -6, -3, 0, 3, 6, \ldots\}$

$[1]_{\text{div by } 3} = \{\ldots, -5, -2, 1, 4, 7, \ldots\}$

$[2]_{\text{div by } 3} = \{-7, -4, -1, 2, 5, 8, \ldots\}$

$\boxed{\text{Th}^m}$ R is an equiv. relation on A. p.215

Given $a, b \in A$ then $[a]_R = [b]_R$ $\boxed{\text{iff}}$ aRb.

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Given $a, b \in A$ then $[a]_R = [b]_R$ $\boxed{\text{iff}}$ aRb.

$\boxed{\text{pf}}$ $\boxed{\text{case 1}}$ $[a]_R = [b]_R \to aRb$

$\boxed{\text{case 2}}$ $aRb \to [a]_R = [b]_R$

Scratch

case 1 ?

$$([a]_R = [b]_R) \to aRb$$

goal!

set set

Note:

$$[a]_R = \{e \mid \boxed{aRa}\}$$

$$[b]_R = \{e \mid eRb\}$$

wht $a \in \{e \mid eRb\}$

goal!