Math 415

Q's properties of $R = \{(a,b) \mid a \equiv_n b\}$

w/ **Def:** $a \equiv_n b$ where $n \mid (b-a)$

or $a = b + kn$

or ⊛ $\underline{a \bmod n} = \underline{b \bmod n}$

Properties: (1) reflexive $\forall a\,(aRa)$ is true?

for above this is $a \equiv_n a$?

check $n \mid (a-a)$ or $n \mid 0$ **true**

(2) irreflexive $\forall a\,(a \not R a)$ is true?

for above this is $a \not\equiv_n a$ is true?

counter example: $3 \equiv_n 3$ so **not irreflexive**.

(3) symmetric $\forall a,b\,(aRb \to bRa)$ is true?

for above this is $a \equiv_n b \to b \equiv_n a$ is true?

pf By direct proof. Assume $a \equiv_n b$. By definition $n \mid (b-a)$ or $a = b + kn$. This gives $b = a + (-k)n$ and $-k \in \mathbb{Z}$

therefore $n \mid (a-b)$ so $b \equiv_n a$.

$\boxed{\text{or}}$ use $a \underset{n}{\equiv} b$ means $a \bmod n = b \bmod n$

Prove $a \underset{n}{\equiv} b \rightarrow b \underset{n}{\equiv} a$

assume $a \underset{n}{\equiv} b$. Gives $a \bmod n = b \bmod n$

by commutativity of equality $b \bmod n = a \bmod n$

so $b \underset{n}{\equiv} a$, by def.

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(4) antisymmetric $\forall a,b\ (aRb \wedge bRa \rightarrow a=b)$ $\boxed{\text{No}}$

for above this is $a \underset{n}{\equiv} b \wedge b \underset{n}{\equiv} a \rightarrow a=b$

counter example: $3 \underset{3}{\equiv} 6 \wedge 6 \underset{3}{\equiv} 3$ and $3 \neq 6$

or $x \underset{n}{\equiv} x+n \wedge x+n \underset{n}{\equiv} x$ but $x \neq x+n$

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(5) asymmetric $\forall a,b\ (aRb \rightarrow b \not R a)$ $\boxed{\text{No}}$

counter example: $x \underset{n}{\equiv} x+n \wedge x+n \underset{n}{\equiv} x$

or $6 \underset{5}{\equiv} 1 \wedge 1 \underset{5}{\equiv} 6$

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(6) transitive $\forall a,b,c\ (aRb \wedge bRc \rightarrow aRc)$

Means $a \underset{n}{\equiv} b \wedge b \underset{n}{\equiv} c \rightarrow a \underset{n}{\equiv} c$

11.3 (2)

2. Let $A = \{a,b,c,d,e\}$. Suppose R is an equivalence relation on A. Suppose R has two equivalence classes. Also aRd, bRc and eRd. Write out R as a set.

$[e]_R = \{ x \mid x \sim e \}$

Notation: if R is an equiv. relation ... you can replace the symbol 'R' with '$\sim$'

so xRe is $x \sim e$

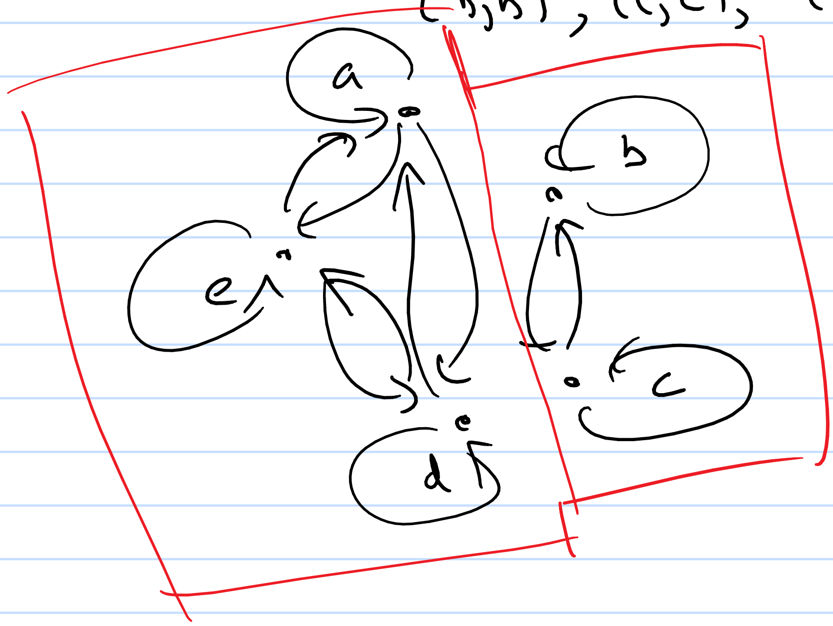
Use thm: $a \sim b$ iff $[a]_\sim = [b]_\sim$

#2 $a \sim d$ and $e \sim d$ and $b \sim c$

$[a] = \{a, d, e\} = [d] = [e]$

$[b] = \{b, c\} = [c]$

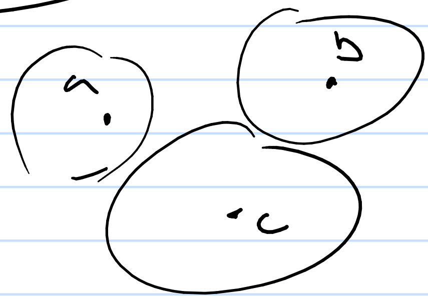
$R = \{ (a,a), (a,d), (d,a), (a,e), (e,a), (d,d), (e,e), (d,e), (e,d), (b,b), (c,c), (b,c), (c,b) \}$



$\boxed{11.4(2)}$

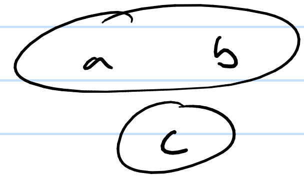
$A = \{a, b, c\}$

)



partition #1

$[a] = \{a\}$
$[b] = \{b\}$
$[c] = \{c\}$

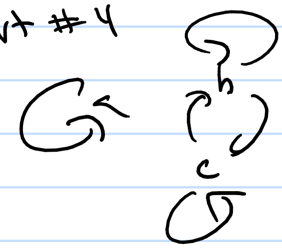


part #2

$[a] = \{a, b\} = [b]$
$[c] = \{c\}$

part #3

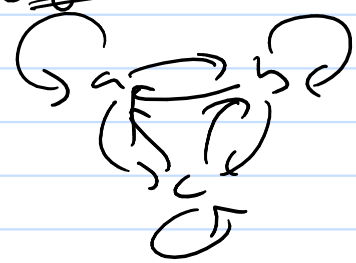
part #4



part #5

a) digraph

digraph of R



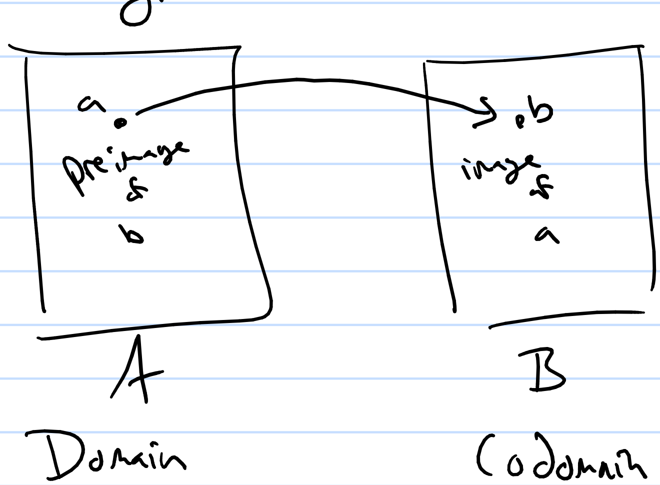
a) partition

① A Relation on a set is a subset of $A \times A$

② A Relation from A to B is a subset of $A \times B$

Arrow Diagram

$|A \times B| = |A|\,|B|$
all possible ordered pairs

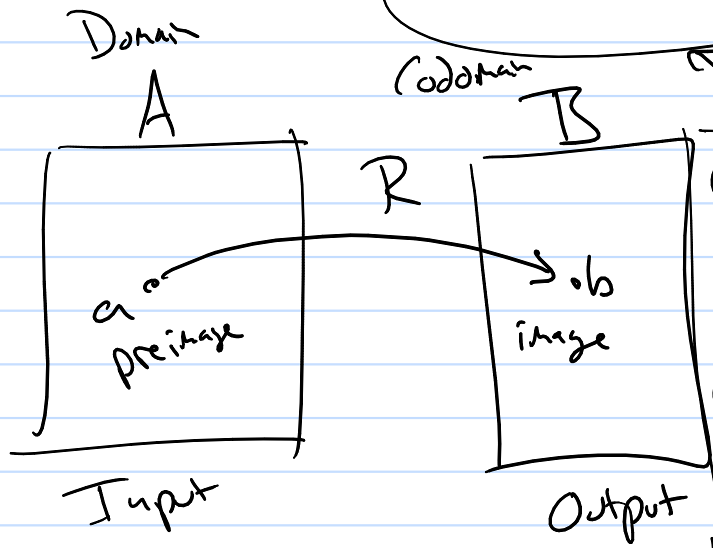


$(a, b) \in R$
or
aRb
or
$R(a) = b$

so

$|\text{Relations}| = 2^{|A||B|}$

$R : A \to B$ is a relation from A to B and is a subset of $A \times B$. ← set of ordered pairs



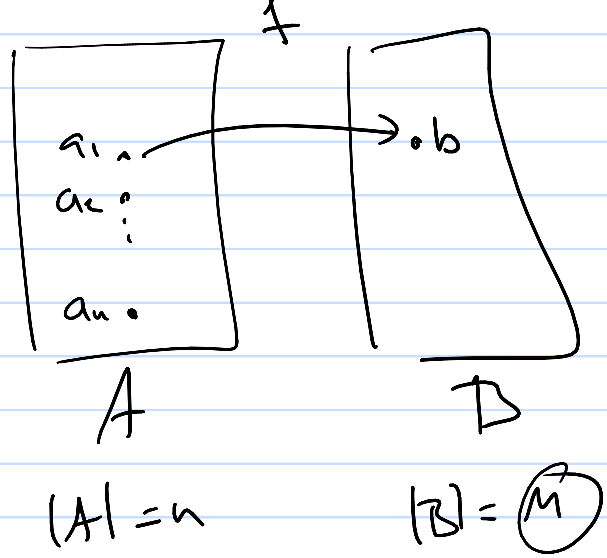
① Restrict R to only ones that use all of domain.

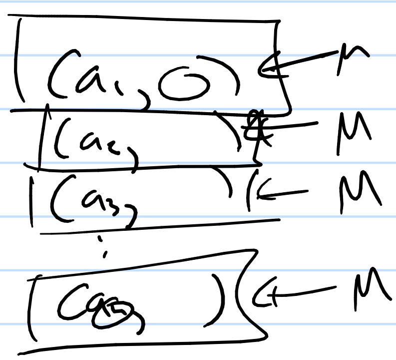
② Restrict R to only ones that for each pre-image it has one image.

All such relations are functions

Def $f \subseteq A \times B$ such that for every $a \in A$, f contains exactly one ordered pair (a,b). Notation: $a f b$ is $f(a) = b$.

Note: $|f| = |B|^{|A|}$ (how many functions $f : A \to B$)



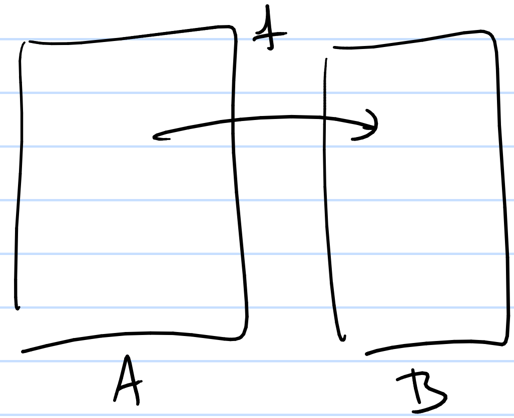


A : domain

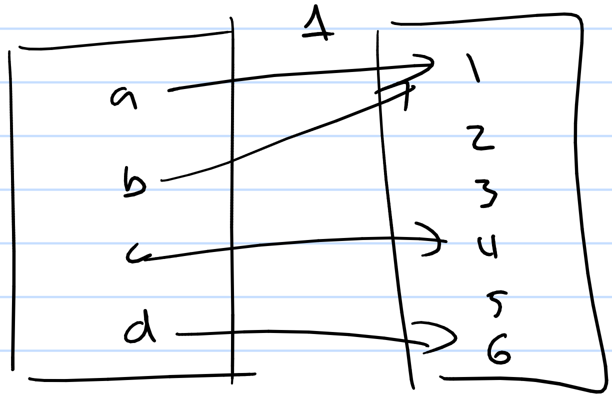
B : codomain

Range $= \{c \mid \exists a \in A, f(a) = c\}$

all elements in B that have a pre-image.



Ex



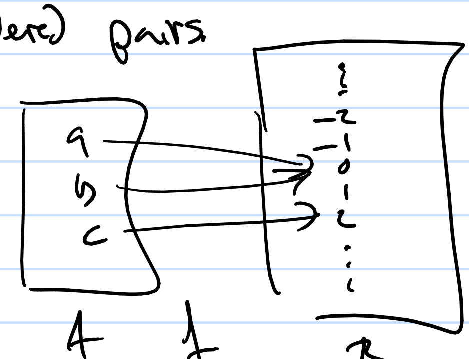
→ Domain: $\{a, b, c, d\}$
Codomain: $\{1, 2, 3, 4, 5, 6\}$
Range: $\{1, 4, 6\}$

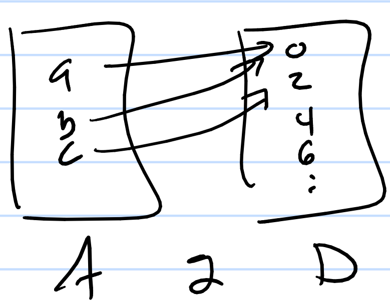
$f = \{(a,1), (b,1), (c,4), (d,6)\}$

1st

$f: A \to B$, $g: A \to D$

what would $f = g$ mean? b/c f and g are simply sets of ordered pairs but $f = g$ mean equal sets of ordered pairs.





$\boxed{f = g}$

Types

① Injective (one-to-one)

$\forall a,b \in A \quad f(a)=f(b) \rightarrow a=b$

$\forall a,b \in A \quad a \neq b \rightarrow f(a) \neq f(b)$

② Surjective (onto)

$\forall b \in B \quad \exists a \in A \quad f(a) = b$

or

(Range = Codomain)

③ bijective (one-to-one $\boxed{\text{and}}$ onto)