Math 415

$\boxed{Q's}$ $\boxed{11.4}$ **3.** Describe the partition of $\mathbb{Z}$ resulting from the equivalence relation $\equiv$ (mod 4).

$$R = \{ (a,b) \mid a \underset{4}{\equiv} b \}$$

$\boxed{\text{in general}}$ $$R = \{ (a,b) \mid a \underset{n}{\equiv} b \}$$

Note: ① $a \underset{n}{\equiv} b$ is
① $n \mid (b-a)$
② $a = b + kn \quad k \in \mathbb{Z}$
③ $a \bmod n = b \bmod n$

1 to 3 are logically equivalent.

② $[e]_R = \{ a \mid eRa \}$

③ a partition of a set is a collection of non-empty disjoint sets whose union is the original set.

ex: if ① $P_i \cap P_j = \phi$ for $i \neq j$

② $\forall i \; P_i \neq \phi$

③ $P_1 \cup P_2 \cup \cdots \cup P_n = S$

then $P_1, P_2, \ldots, P_n$ is a partition of S.

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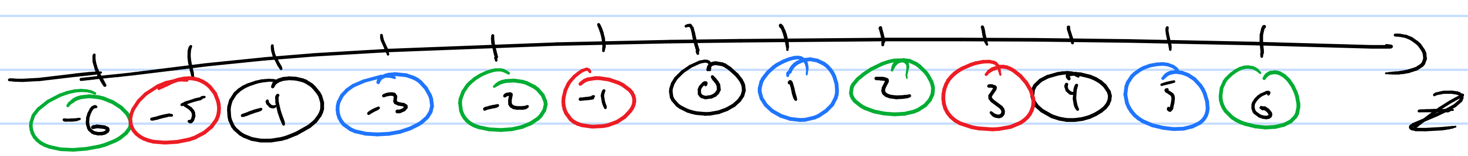
for $R = \{ (a,b) \mid a \underset{4}{\equiv} b \}$ what is its equiv. class?



$$[11]_R = \{ a \mid a \underset{4}{\equiv} 11 \} = \{ \ldots, -5, -1, 3, 7, 11, 15, 19, \ldots \}$$

$$[20]_R = \{ a \mid a \underset{4}{\equiv} 20 \} = \{ \ldots, -8, -4, 0, 4, 8, 12, 16, 20, \ldots \}$$

12.1

8. Consider the set $f = \{(x,y) \in \mathbb{Z} \times \mathbb{Z} : x + 3y = 4\}$. Is this a function from $\mathbb{Z}$ to $\mathbb{Z}$? Explain.

Domain

Codomain

Def Relation on set A. R is a subset of $A \times A$.

but $A \times A = \{ (a,b) \mid a \in A \wedge b \in A \}$

Def Function ① It is a relation

② all elements in domain map to codomain

③ each element in domain has exactly one element of codomain.

$f = \{ (x,y) \mid x \in \mathbb{Z} \wedge y \in \mathbb{Z} \wedge x + 3y = 4 \}$

ex $(1,1) \in f$

b/c 0 has no image

So f is not a function.

f let's find $(x,y) \in f$

$0 \to \times$

$1 \to 1$ $(1,1) \in f$

$2 \to \times$

$3 \to \times$

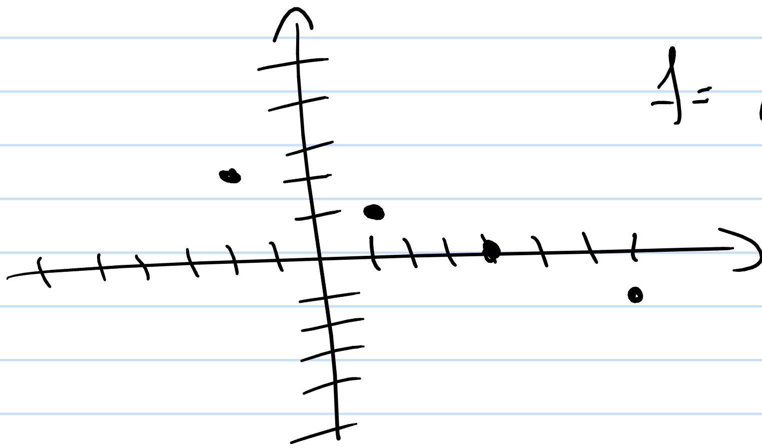
$4 \to 0$ $(4,0) \in f$

ex $f = \{ (x,y) \mid x \in \mathbb{Z} \wedge y \in \mathbb{Z} \wedge y + 3x = 4 \}$

$f = \{ (x,y) \mid x \in \mathbb{Z} \wedge y \in \mathbb{Z} \wedge y = 4 - 3x \}$

is a function from $\mathbb{Z}$ to $\mathbb{Z}$.

$x+3y=4$ or $y = \frac{4-x}{3}$ on $\mathbb{Z}$ to $\mathbb{Z}$



$f = \{(x,y) \mid x+3y=4\}$ on $\mathbb{Z}$ to $\mathbb{Z}$

Exam 4 take home Wed to Friday (hand in online)

$\boxed{\text{11 probs}}$

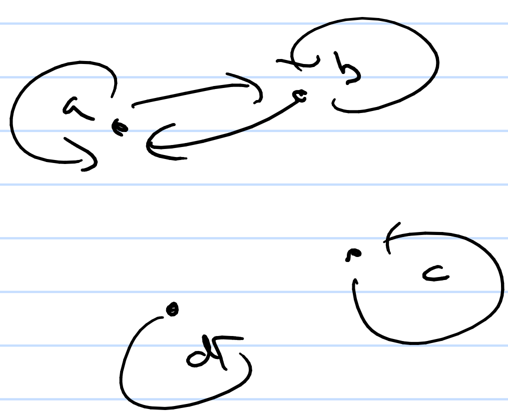
Relations (ch 11)

(1)
(2) } prove or disprove each property for a relation

ex R on $A = \{a, b, c, d\}$

as list? $R = \{(a,a), (b,b), (c,c), (d,d), (a,b), (b,a)\}$

as digraph?



(3) Is a given relation an equiv. relation?

(4) Given an equiv. relation, state/describe its equiv. classes.

(ex) The relation on polynomials such that..

$$R = \{ (p_1, p_2) \mid \frac{d^2}{dx^2}[p_1] = \frac{d^2}{dx^2}[p_2] \}$$

is an equiv. relation.

$\boxed{\text{Describe its equiv. classes.}}$

play around

$$(2x^2+1, 2x^2+x) \in R$$

b/c $(2x^2+1)'' = 4$

$(2x^2+x)'' = 4$

$$[2x^2+1]_R = \{ p(x) \mid p(x) = 2x^2 + ax + b \wedge a, b \in \mathbb{R} \}$$

$$[4x^{101} + 2x^{50} - 3x^4 + x^2 + 2x - 1]_R$$
$$= \{ p(x) \mid p(x) = 4x^{101} + 2x^{50} - 3x^4 + x^2 + ax + b \wedge a, b \in \mathbb{R} \}$$

(5) p. 215 <u>thm 11.1</u> & BoP

Rewrite this proof for a student who is <u>bad</u> @ set notation and relation notation and equiv. classes.

ch 12 Functions

(6) Is a given relation a function?

(7) Is a given relation a function?

(8) Prove or disprove Injection.

(9) Prove or disprove Surjection

(10) Prove or disprove bijection

(11) How many? (1) relations
(2) functions
(3) Injections
(4) Surjections
(5) bijections

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ch 14 Countable / Uncountable.

(1) Cardinality of finite sets.

$|S|$ = number of unig. elements.

<u>Finite:</u> $|S| = n$, such that $n \in \{0, 1, 2, ..\}$

So it is easy to understand ..

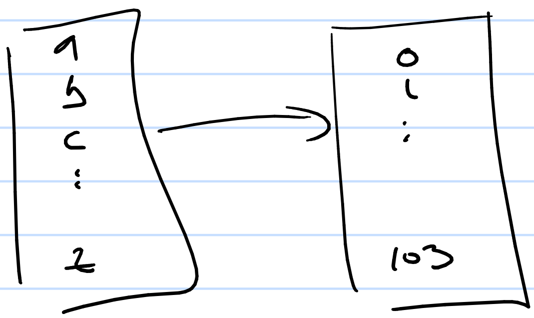
① $|\{0, 1, 2, .., 103\}| = 104$

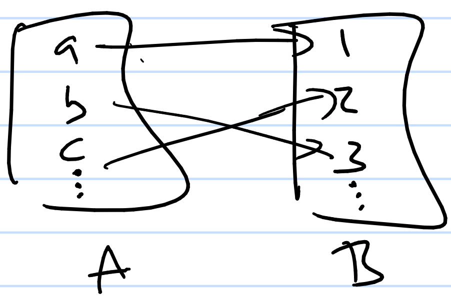
② $|\{a, b, c, .., z\}| = 26$

③ b/c $26 < 104$ $|\{a, b, c, .., z\}| < |\{0, 1, 2, .., 103\}|$

④ $|\{a, b, c, .., z\}| = |\{-2, -1, 0, 1, 2, .., 23\}|$

So





If you have a bijection then $|A| = |B|$