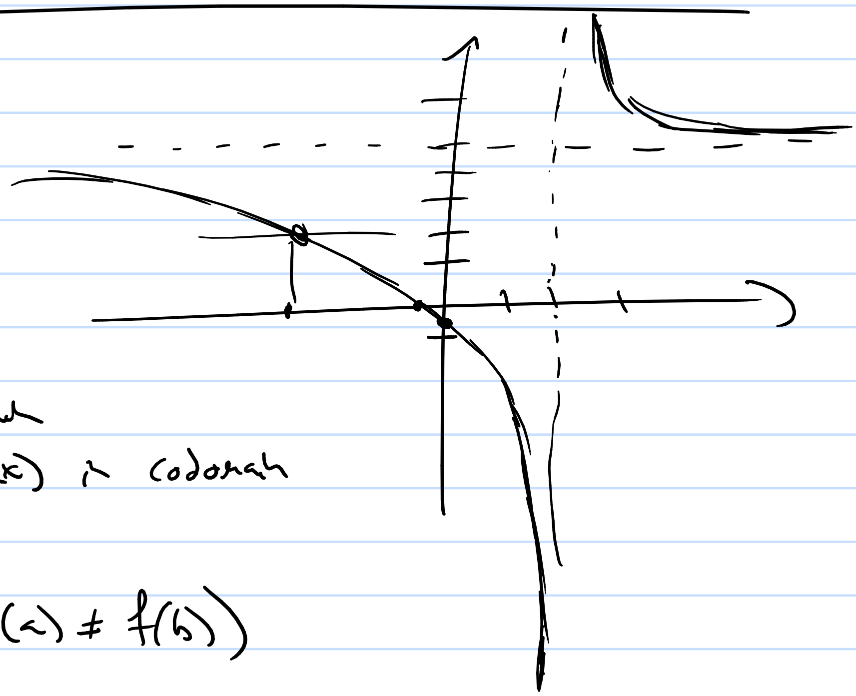
Math 415

$\boxed{\text{Q's}}$ $f(x) = \dfrac{5x+1}{x-2}$



$f: \mathbb{R} - \{2\} \to \mathbb{R} - \{5\}$

Injective: every x in domain
has unique $f(x)$ in codomain

logical $\forall\, a,b \in$ domain
$(a \neq b \to f(a) \neq f(b))$

or

$\forall\, a,b \in$ domain
$(f(a) = f(b) \to a = b)$

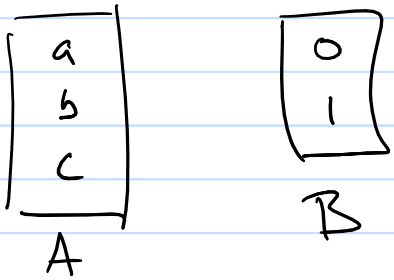
a,b are reals that are not 2.

assume $\dfrac{5a+1}{a-2} = \dfrac{5b+1}{b-2}$

$(5a+1)(b-2) = (5b+1)(a-2)$

goal
$a = b$

4. There are eight different functions $f : \{a, b, c\} \to \{0, 1\}$. List them. Diagrams suffice.



function w/ arrows ??

restrictions

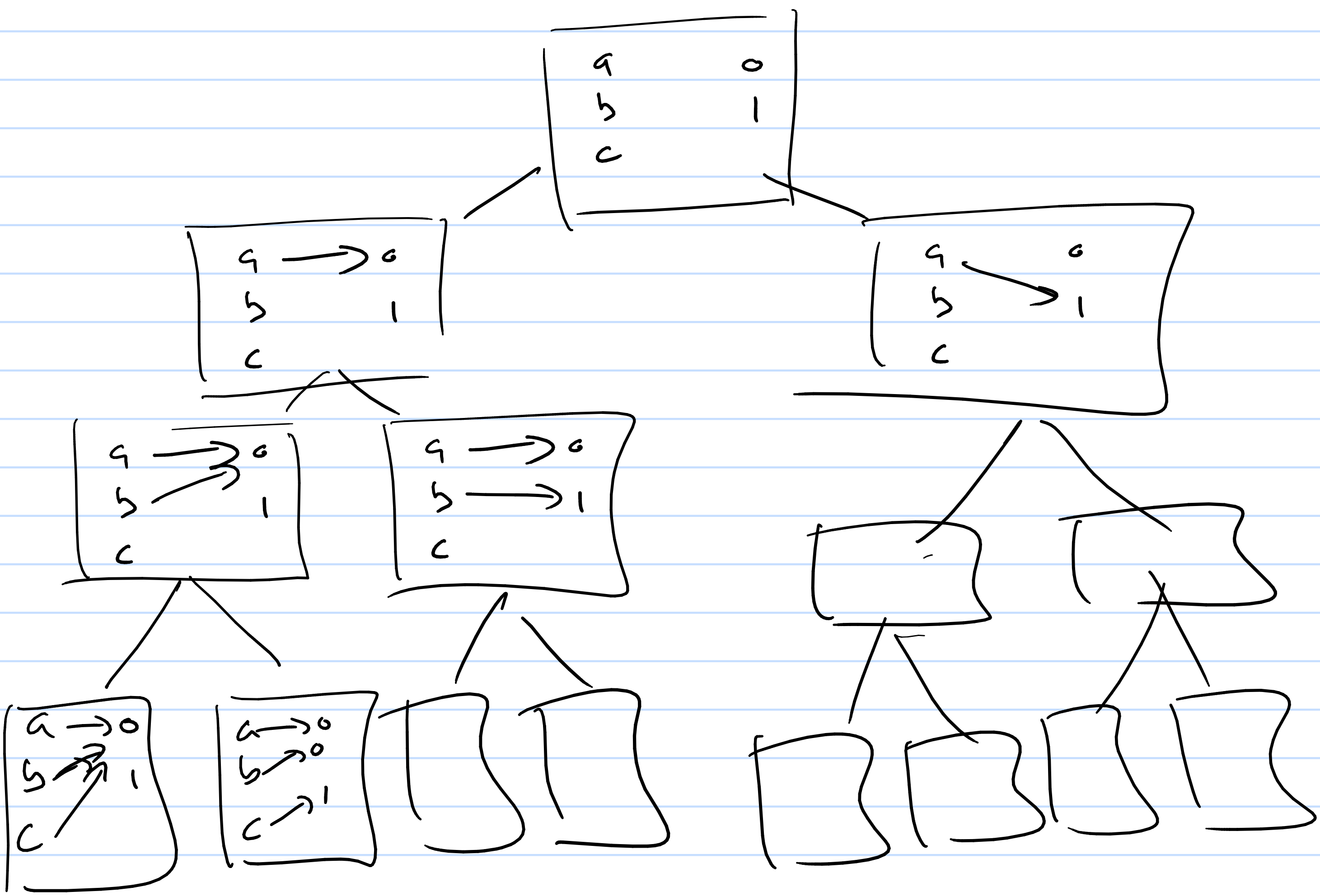
① one in A goes to one in B

② all in A go somewhere

to make a function we need 3 arrows each can go to 2 possible locations.

Make arrow 1 (2 ways)
and " " 2 (2 ways)
and " " 3 (2 ways)

$2 \cdot 2 \cdot 2 = 2^3 = 8$



Countability (listability)

① Finite $|A| = n \in \{0, 1, 2, 3, \ldots\}$

and $|\phi| = 0$

② Not finite $|A|$ is not a finite number.

ex $|\{1, 2, 3, \ldots\}|$ not finite

$|\mathbb{Q}|$ not finite

$|\mathbb{R}|$ not finite

$|\mathbb{C}|$ not finite

$|\{\ldots, -2, -1, 0, 1, 2, \ldots\}|$ not finite

Focus on $\{1, 2, 3, \ldots\}$ $\boxed{\text{and}}$

if you can find a bijection between A and B

$|A| = |B|$

call $|\{1, 2, 3, \ldots\}| = \aleph_0$ (aleph null)

Def: $|S| = \aleph_0$ we call S to be countable or listable.

$S = \{s_1, s_2, s_3, \ldots\}$

Hotel $\quad r_1, r_2, r_3, r_4, r_5, r_6, \dots \quad \{r_i\}$

Math club $\quad m_1, m_2, m_3, m_4, \dots \quad \{m_j\}$

$m_1 \to r_1$

$m_2 \to r_2$

$m_3 \to r_3$

$\vdots$

$$|\text{Hotel}| = |\text{Math club}| = \aleph_0$$

$$\begin{array}{ccc} r_1, & r_2, & \dots \\ m_1 & m_2 & \end{array}$$

Stat club $\quad s_1, s_2, s_3, \dots$

$$\begin{array}{ccccccc} r_1, & r_2, & r_3, & r_4, & r_5, & r_6, & \dots \\ s_1 & m_1 & s_2 & m_2 & s_3 & m_3 & \dots \end{array}$$

$$m_j \to r_{2j}$$

$$s_j \to r_{2j-1}$$

$$|\text{Hotel}| = |\text{Math} \cup \text{Stat}| = \aleph_0$$

$$|\text{Hotel}| = |\text{Math}| = \aleph_0$$

$$|\text{Hotel}| = |\text{Stat}| = \aleph_0$$

$$\begin{array}{cccc} P_{1,1} & P_{1,2} & P_{1,3} & \dots \\ P_{2,1} & P_{2,2} & P_{2,3} & \dots \\ P_{3,1} & P_{3,2} & P_{3,3} & \dots \end{array}$$

$P_{11}, P_{21}, P_{31}, P_{32}, P_{22}, P_{12}, \dots$

$i \times j$

$$\begin{array}{c|ccccc}
 & P_{11} & P_{12} & P_{13} & P_{14} & \cdots \\
2 \nearrow & P_{21} & P_{22} & P_{23} & P_{24} & \cdots \\
3 \nearrow & P_{31} & P_{32} & P_{33} & P_{34} & \cdots \\
4 \nearrow & P_{41} & P_{42} & P_{43} & P_{44} & \cdots \\
5 \nearrow & \vdots & \vdots & \vdots & \vdots &
\end{array}$$

P_{ij}

so $|\{P_{ij}\}| = \aleph_0$

$$\begin{array}{cccccc}
\pm\frac{1}{1} & \pm\frac{2}{1} & \pm\frac{3}{1} & \pm\frac{4}{1} & \pm\frac{5}{1} & \cdots \\
\pm\frac{1}{2} & \pm\frac{2}{2} & \pm\frac{3}{2} & \pm\frac{4}{2} & \pm\frac{5}{2} & \cdots \\
\pm\frac{1}{3} & \pm\frac{2}{3} & \pm\frac{3}{3} & \pm\frac{4}{3} & \pm\frac{5}{3} & \cdots \\
\pm\frac{1}{4} & \pm\frac{2}{4} & \pm\frac{3}{4} & \pm\frac{4}{4} & \pm\frac{5}{4} & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots &
\end{array}$$

$|\mathbb{Q}| = \aleph_0$

$|\mathbb{Q}| = |\{1, 2, 3, \dots\}|$

$\mathbb{R} = \mathbb{Q} \cup$ irrationals

$\uparrow$? $\qquad \uparrow$ $|\mathbb{Q}| = \aleph_0$ $\qquad$ ex $\sqrt{2}, \sqrt{3}, \dots$

Assume countable

$[0, 1]$

$$\begin{aligned}
r_1 &= 0.d_{11}\, d_{12}\, d_{13} \dots \\
r_2 &= 0.d_{21}\, d_{22}\, d_{23} \dots \\
r_3 &= 0.d_{31}\, d_{32}\, d_{33} \dots \\
&\vdots
\end{aligned}$$

$d_{ij} = \{0, 1, 2, \dots, 9\}$

Note: $\overline{9}$ is an issue!

$0.29999\dots$
$= 0.30000\dots$

we remove all $\bar{9}$ versions of terminating decimals.

So all $r_i = 0.d_{i1}\, d_{i2}\, d_{i3} \ldots$

have one uniq. decimal representation.

why? $r_a = 0.\_\ \_\ \_\ \_\ 3\ \_\ \_\ \_ \ldots$

$r_b = 0.\_\ \_\ \_\ \_\ 4\ \_\ \_\ \_ \ldots$

different $r_a \neq r_b$

Countable? all r between 0 and 1 (if we exclude $\bar{9}$)

are uniquely here.

$r_1 = 0.d_{11}\, d_{12}\, d_{13} \ldots$

$r_2 = 0.d_{21}\, d_{22}\, d_{23} \ldots$

$\vdots$

Consider: $r^* = 0.d_1 d_2 d_3 \ldots$

pick d_i such that these are not 0, not 9

and d_{ii} of r_i is

$\forall i \quad d_i \neq d_{ii} \quad \forall i\ r^* \neq r_i$

So $|\mathbb{R}| \neq \aleph_0$ not countable.

and $\aleph_0 < |\mathbb{R}| = \aleph_1$

$$|P(\mathbb{Z})| = |\mathbb{R}|$$

$$2^{|\mathbb{Z}|} = 2^{\aleph_0} = \aleph_1$$

<u>Sets</u> <u>sizes</u> transfinites

$$\boxed{0, 1, 2, \dots}, \overbrace{\aleph_0, \aleph_1, \aleph_2, \aleph_3, \dots}$$

finite

$\aleph_0$: $|\{1, 2, 3, \dots\}|$