Math 415

$\boxed{\text{Q's}}$

7) Is the relation $R = \{(s, n) | s$ is any string and n is the number of characters in $s\}$ a function from the set of strings to the set of integers?

the set of strings are of seq of characters of no length to finite length.

null string.

So any string is $c_1 c_2 c_3 \ldots c_n$

ex) S = Mark was here!

('Mark was here!', 14)

10) Give an example of a bijection from the rational numbers to the positive integers.

$f: \mathbb{Q} \to \mathbb{Z}^+$

0

$\pm\frac{1}{1}$	$\pm\frac{2}{1}$	$\pm\frac{3}{1}$	$\pm\frac{4}{1}$
$\pm\frac{1}{2}$	$\pm\frac{2}{2}$	$\pm\frac{3}{2}$	$\pm\frac{4}{2}$
$\pm\frac{1}{3}$	$\pm\frac{2}{3}$	$\pm\frac{3}{3}$	$\pm\frac{4}{3}$
$\pm\frac{1}{4}$	$\pm\frac{2}{4}$	$\pm\frac{3}{4}$	$\pm\frac{4}{4}$

the rationals are all $\frac{a}{b}$ such that $a, b \in \mathbb{Z}$, $b \neq 0$, and a, b have no common factors. The above table contains all rationals with fractions that are not rational. Each diagonal of $a + b = n$, except diagonal $= 1$ is just the rational 0, have ...

$0 \leftrightarrow 1$ | $\frac{1}{2} \leftrightarrow 4$ | $-2 \leftrightarrow 7$

$1 \leftrightarrow 2$ | $-\frac{1}{2} \leftrightarrow 5$ | $\vdots$

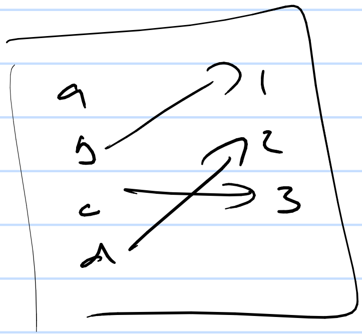
$-1 \leftrightarrow 3$ | $2 \leftrightarrow 6$

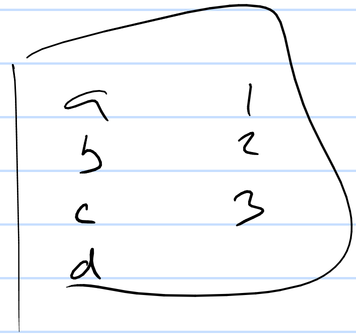
exclude all fractions not in $\mathbb{Q}$.

(11) $|A| = 4$ $\quad |B| = 3$ $\qquad A \times B = \{(a,b) \mid a \in A \wedge b \in B\}$

$|A \times B| = 4 \cdot 3$

(d) Surjection





arrow to 1 (4)

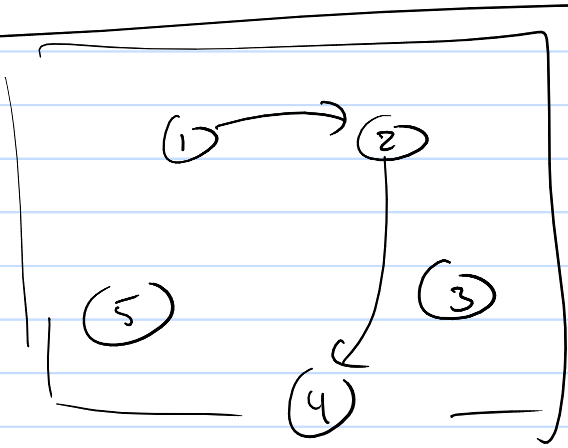
all arrow to 2 (3)

all arrow to 3 (2)

all (3)

7

#2



$R = \{(a,b) \mid b = 2a\}$

$R = \{(1,2), (2,4)\}$

$A = \{1, 2, 3, 4, 5\}$

Reflexive: $\forall a\, (aRa)$ which is $a = 2a$

Counter example $1 \neq 2 \cdot 1$

BoP ch 13 | Due Next Wed

13.1 (thm 13.1 + corollary)

13.2 (2, 4, 6)

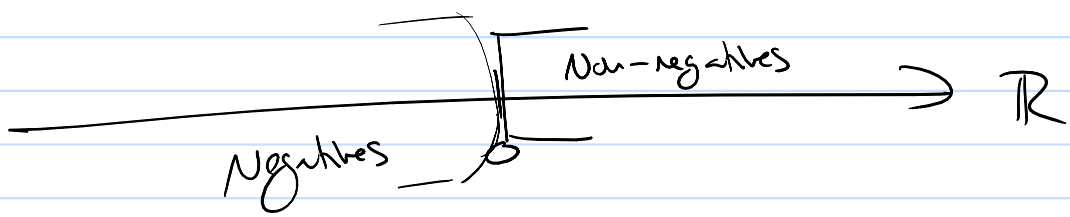
14.1 (3, 4)

ch 14 prove $\mathbb{Q}$ is countable
$\mathbb{R}$ is uncountable

Proofs in Calculus

$x, y \in \mathbb{R}$

triangle inequality — 1 variation: $|x+y| \le |x| + |y|$



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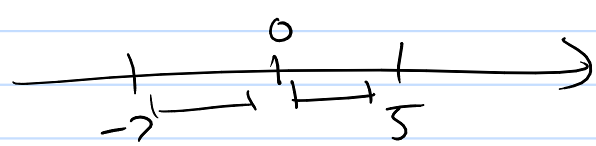
thm for $x, y, z \in \mathbb{R}$

$$|x-y| \le |x-z| + |z-y|$$

Scratch: what do we see?

① $|\text{var}_1 - \text{var}_2|$ = positive distance between var_1, var_2

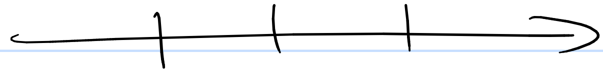
(ex)



$$|-7-5| = 12$$

$$|5-(-7)| = 12$$

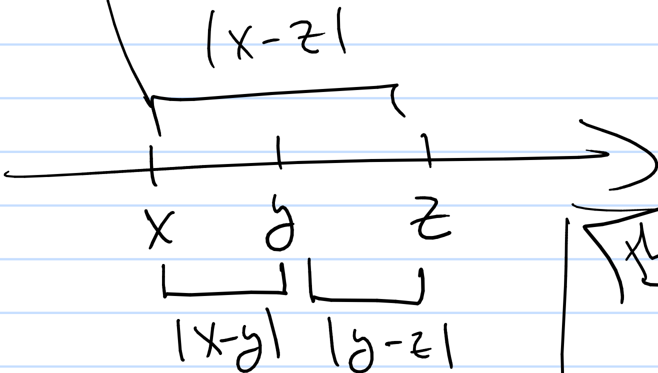
So $x, y, z \in \mathbb{R}$



$x <$: $x<y$, $x<z$ → $x<y<z$, $x<z<y$

$y <$: $y<x$, $y<z$ → $y<x<z$, $y<z<x$

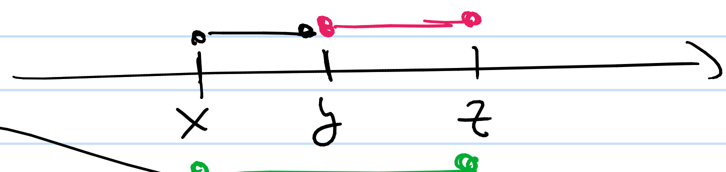
$z <$: $z<x$, $z<y$ → $z<x<y$, $z<y<x$

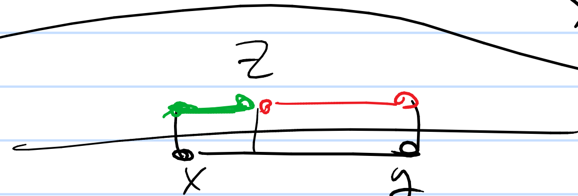


Th△ $|x-y| \le |x-z| + |z-y|$

Pf: For $x, y, z \in \mathbb{R}$ there will be six cases for them to be placed on the real number line.

Case 1 x, y, z





Corollaries

given $|x-y| \le |x-z| + |z-y|$ is true $\forall x, y, z$

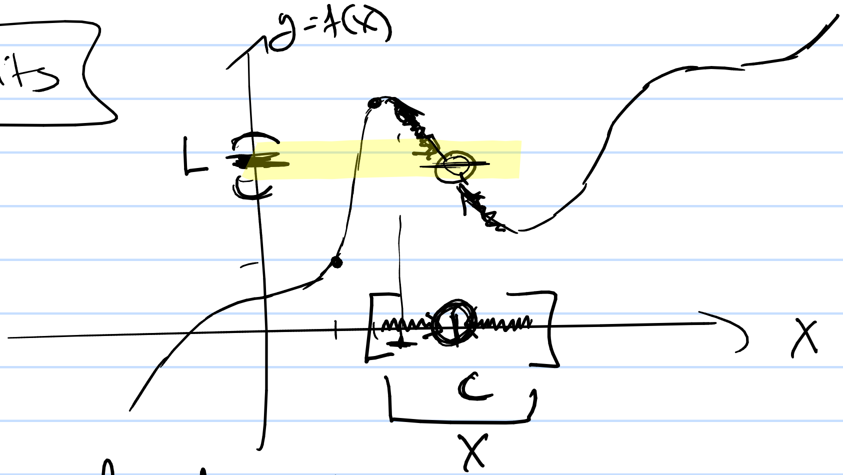
(1) let $z=0$ $\qquad |x-y| \le |x| + |y|$

(2) let $z=0, y=-y$ $\qquad |x+y| \le |x| + |y|$

(3) let $z=-y, y=0$ $\qquad |x| \le |x+y| + |-y|$

$|x| - |y| \le |x+y|$

Limits



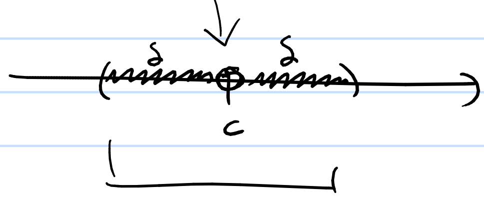
Def: $\lim_{x\to c} f(x) = L$

Suppose $f: X \to \mathbb{R}$, $X \subseteq \mathbb{R}$ and $c \in \mathbb{R}$

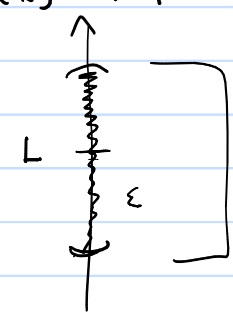
Then $\lim_{x\to c} f(x) = L$ means for any real number $\varepsilon > 0$,

there is a real $\delta > 0$ for which...

if $0 < |x-c| < \delta$, then $|f(x) - L| < \varepsilon$



ball around c & radius δ and not including the center



ball around L of radius ε.