

Math 511

Monday: 1.1 (1b, 5c, 6e, 8*, 9*, 10, 11*)

9 $\left[\begin{array}{l} -M_1 x_1 + x_2 = b_1 \\ -M_2 x_1 + x_2 = b_2 \end{array} \right]$ Solve?

Solnra $\rightarrow \begin{cases} 1 & (\text{consistent}) \\ \infty \end{cases}$

$x_2 = b_1 + M_1 x_1$ no solnra (inconsistent)

so, $\left[-M_2 x_1 + (b_1 + M_1 x_1) = b_2 \right]$

$$-M_2 x_1 + b_1 + M_1 x_1 = b_2$$
$$M_1 x_1 - M_2 x_1 = b_2 - b_1$$

$\otimes (M_1 - M_2) x_1 = b_2 - b_1$ $x_2 = ?$

$x_1 = \frac{b_2 - b_1}{M_1 - M_2}$

10 if $M_1 = M_2$ \otimes is $0 x_1 = b_2 - b_1$

$$0 = b_2 - b_1$$

$$\begin{array}{l} -M_1 x_1 + x_2 = b_1 \\ -M_2 x_1 + x_2 = b_2 \end{array}$$

$\wedge M_1 = M_2 \stackrel{\text{all it}}{=} M$

$$\left[\begin{array}{l} -M x_1 + x_2 = b_1 \\ -M x_1 + x_2 = b_2 \end{array} \right]$$

$$\begin{array}{l} 3x = 1 \\ 3x = -1 \end{array}$$

Solving Systems of Equations w/ Augmented Matrices.

Fact of Elimination / Substitution

$$\begin{cases} 2(3x - y = 2) \\ x + 2y = 4 \end{cases} \rightarrow \begin{cases} 6x - 2y = 4 \\ x + 2y = 4 \end{cases}$$

$$\underline{2r_1 + r_2} \rightarrow \begin{cases} 7x = 8 & \text{New } r_1 \\ x + 2y = 4 \end{cases}$$

#1 ok? Mult. by non-zero

#2 ok? New row = linear combination of two rows.

#3 ok? Rearrange rows. (Row Swap)

$$\text{(cr)} \quad \begin{cases} 2x - y + z = 3 \\ x - 4y + 2z = 1 \\ 7x - 11y + 2z = 3 \end{cases} \rightarrow \begin{cases} x - 4y + 2z = 1 \\ 2x - y + z = 3 \\ 7x - 11y + 2z = 3 \end{cases}$$

$$\text{(cr)} \quad \begin{cases} x - y + z = 1 \\ 2x + y - 3z = 2 \\ 3x + 2y + z = 3 \end{cases} \quad \begin{cases} -2r_1 + r_2 = N r_2 \\ -3r_1 + r_3 = N r_3 \end{cases} \quad \begin{cases} x - y + z = 1 \\ 3y - 5z = 0 \\ 5y - 2z = 0 \end{cases}$$

$$-\frac{2}{3}r_2 + r_3 = N r_3$$

$$\begin{cases} x - y + z = 1 \\ 3y - 5z = 0 \\ \underline{\quad \quad \quad} \end{cases}$$

Per Augmented Matrix

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 2 & 1 & -3 & 2 \\ 3 & 2 & 1 & 3 \end{array} \right]$$

row ops & elimination became

row ops & Aug. Matrices

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 2 & 1 & -3 & 2 \\ 3 & 2 & 1 & 3 \end{array} \right]$$

$$\begin{array}{l} -2r_1 + r_2 \\ -3r_1 + r_3 \end{array}$$

$$\begin{array}{c} x \quad y \quad z \quad \text{const} \\ \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 3 & -5 & 0 \\ 0 & 5 & -2 & 0 \end{array} \right] \end{array}$$

Goal

Triangular Matrices

use row ops until you get zeros below
lead variables... row echelon form

↓ 1st non-zero

(ex) →

$$\left[\begin{array}{ccccc|c} 1 & 2 & 1 & 3 & 2 & 1 & 4 \\ 0 & 0 & 2 & -1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 & 2 & 5 \end{array} \right]$$

$x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5$

Steps to make
this

Gaussian
elimination

Quiz

Solve

$$\begin{aligned} 2y + 2u &= 8 \\ x - 2y + z + u &= 0 \\ 2x + y + z - u &= 3 \\ x + y + u &= 5 \end{aligned}$$

#9

$$\left[\begin{array}{ccc|cc} 0 & 2 & -2 & 1 & 4 \\ 1 & -2 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 3 \\ 1 & 1 & 0 & 0 & 5 \end{array} \right]$$