Math 511

Q's 1.1 (10)

$$\left.\begin{array}{l} a_{11}x_1 + a_{12}x_2 = 0 \\ a_{21}x_1 + a_{22}x_2 = 0 \end{array}\right\}$$

$x_1 = 0$ and $x_2 = 0$

$a_{11}0 + a_{12}0 \overset{?}{=} 0$

$a_{21}\cdot 0 + a_{22}\cdot 0 \overset{?}{=} 0$

$\boxed{\text{consistent}}$ → solution → exactly 1
→ ∞ solutions

$\boxed{1.2}$ **Def:**

m-eqn's

$$\left[\begin{array}{l} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = 0 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = 0 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = 0 \end{array}\right.$$

n-unknowns

all zero's? if yes... call this a homogeneous system.

$\boxed{1.2}$ **Note:** all homogeneous systems have at least the all zero soln. $x_1 = 0, x_2 = 0, \ldots, x_n = 0$

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for #10 we see $x_1 = 0$, $x_2 = 0$ is a solution

$$\left[\begin{array}{l} a_{11}\cdot 0 + a_{12}\cdot 0 = 0 \\ a_{21}\cdot 0 + a_{22}\cdot 0 = 0 \end{array}\right.$$

So consistent

Due Friday: 1.2(1, 2, 3, 6, 7*, 8*, 9, 15)

ex 3

$$\begin{cases} 3x_1 + x_2 - x_3 = 0 \\ x_1 - x_2 + 2x_3 = 0 \end{cases}$$

$$\left[\begin{array}{ccc|c} 3 & 1 & -1 & 0 \\ 1 & -1 & 2 & 0 \end{array}\right]$$

swap r_1, r_2

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 3 & 1 & -1 & 0 \end{array}\right]$$

$-3r_1 + r_2 = N r_2$

$$\begin{array}{cccc} x_1 & x_2 & x_3 & \\ \end{array}$$
$$\left[\begin{array}{ccc|c} \boxed{1} & -1 & 2 & 0 \\ 0 & \boxed{4} & -7 & 0 \end{array}\right]$$

no lead var.
call x_3 free

call $\boxed{x_3 = \alpha}$

back solve:

$4x_2 - 7x_3 = 0$

$4x_2 - 7\alpha = 0$ so $\boxed{x_2 = \frac{7}{4}\alpha}$

$x_1 - x_2 + 2x_3 = 0$

$x_1 - \frac{7}{4}\alpha + 2\alpha = 0$ so $\boxed{x_1 = -\frac{1}{4}\alpha}$

ans $\boxed{\left(-\frac{1}{4}\alpha, \frac{7}{4}\alpha, \alpha\right)}$

1.3/1.4 Matrix Arithmetic / Algebra

Matrix: rectangular group of numbers / scalars.

ex $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ 2 x 3 Matrix

row col.

$\boxed{A = [a_{ij}]}$

matrix scalars

ex $a_{22} = 5$

$a_{13} = 3$

Special Matrices: 1 x n call it a row vector

m x 1 call it a column vector.

ex $\vec{v} = [1\ 2\ 0\ 1\ -1]$ row vector 1 x 5

$\mathbf{v} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ column vector 3 x 1

bold font lower case is a vector

$I = \begin{bmatrix} 1 & \ddots & 0 \\ 0 & & 1 \end{bmatrix}$ ex $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$O = [0_{ij}]$ ex $O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

Arithmetic

① **Equal** $A = B$ (means?)

if $A = [a_{ij}]$ $B = [b_{ij}]$

that both A, B are $m \times n$ (same size)

and $a_{ij} = b_{ij}$ for all i, j positions.

② **transpose** $A^T =$ A with row, col. swapped.

ex $\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$

③ $A \pm B = [a_{ij} \pm b_{ij}]$ both are $m \times n$

$(2x + 3y) + (x - 4y) = 3x - y$

$[2 \;\; 3] + [1 \;\; -4] = [3 \;\; -1]$

④ $\alpha A = [\alpha a_{ij}]$ $\boxed{\text{Scalar multiplication}}$

$\underset{\text{Scalar}}{\alpha} \; \underset{\text{matrix}}{A}$

Distribution: $3(2x + y) = 6x + 3y$

⑤ $A - B = A + (-1)B = [a_{ij} - b_{ij}]$

Multiplication?

Matrix · Matrix = ?

ex $3x + 2y = 1$

coef: $[3 \quad 2]$ vars? x, y

other side $= 1$ is a scalar!

$$\begin{bmatrix} 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 1$$

scalar product — row vector · col. vector = scalar

Inner product.

$$3 \cdot x + 2 \cdot y = 1$$