Math 511

Due Monday
1.3(2, 4c, 8*, 9, 10b, 11, 12, 15, 16*)

Q's

$$\begin{cases} 3x + y = 2 \\ x + y = 4 \\ 4x + 2y = 6 \\ 2x + 2y = 8 \end{cases}$$

over det

$$\left[\begin{array}{cc|c} 3 & 1 & 2 \\ 1 & 1 & 4 \\ 4 & 2 & 6 \\ 2 & 2 & 8 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 1 & 4 \\ 3 & 1 & 2 \\ 4 & 2 & 6 \\ 2 & 2 & 8 \end{array}\right] \quad \begin{array}{l} r_2 - 3r_1 = Nr_2 \\ r_3 - 4r_1 = Nr_3 \\ r_4 - 2r_1 = Nr_4 \end{array} \quad \left[\begin{array}{cc|c} 1 & 1 & 4 \\ 0 & -2 & -10 \\ 0 & -2 & -10 \\ 0 & 0 & 0 \end{array}\right] \quad r_3 - r_2 = Nr_3$$

$$\left[\begin{array}{cc|c} 1 & 1 & 4 \\ 0 & -2 & -10 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right] \qquad \left[\begin{array}{cc|c} \boxed{1} & 1 & 4 \\ 0 & \boxed{1} & 5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right] \text{ — def.}$$

$y = 5$

$x = -1$

$\boxed{(-1, 5)}$

1.3/1.4

Matrix (capital letters to represent)

Vector (bold font lower case to represent)

ex B is a matrix

$\mathbf{v}$ is a column vector

$\vec{v}$ is a row vector

ops on Matrix / Vectors

① Equality $A = B$ if and only if $a_{ij} = b_{ij}$

② A^T swap rows and cols. if $A = [a_{ij}]$ $B = [b_{ij}]$

v^T = is a row vector

$\vec{v}^T$ = is a col. vector

③ - ④ - ⑤ $A + B$

αA ← scalar multiplication

$A - B$

Matrix · Matrix ← Matrix Multiplication

A is $m \times n$

rows cols

row vector · col vector = Scalar

scalar product (eventually we will have a concept like this: inner product)

ex $$[1 \;\; -1 \;\; 0 \;\; 2] \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix} = (1)(-1) + (-1)(1) + (0)(-1) + (2)(1)$$

$$= 0$$

1×4

$\overset{?}{=}$

4×1

ex $3x + 4y - z = 2$ $$[3 \;\; 4 \;\; -1] \begin{bmatrix} x \\ y \\ z \end{bmatrix} = [2]$$

ⓔⓧ $\begin{bmatrix} -1 & 3 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = -1$

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attempt #2

Matrix · col. vector

$\boxed{3x - 2y + z} = 1$

$x + y + 2z = 2$

$$\begin{bmatrix} \boxed{3 \quad -2 \quad 1} \\ 1 \quad\ \ 1 \quad\ \ 2 \end{bmatrix} \cdot \begin{bmatrix} \boxed{\begin{matrix} x \\ y \\ z \end{matrix}} \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Note:

$$\underset{m \times n}{A} = [a_{ij}] = \begin{array}{c} \left[\begin{matrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{matrix}\right] \begin{matrix} \to \vec{a}_1 \\ \to \vec{a}_2 \\ \\ \\ \end{matrix} \\ \begin{matrix} a_1 & \quad a_2 & \qquad\quad a_n \end{matrix} \end{array}$$

$$A = \begin{bmatrix} \vec{a}_1 \\ \vec{a}_2 \\ \vec{a}_m \end{bmatrix} \quad \text{or}$$

$$A = \begin{bmatrix} a_1 & a_2 & \cdots & a_n \end{bmatrix}$$

$$\text{so} \quad \underset{\uparrow}{A}\ \underset{\uparrow}{v} = \begin{bmatrix} \vec{a}_1 \\ \vec{a}_2 \\ \vdots \\ \vec{a}_m \end{bmatrix} v = \begin{bmatrix} \vec{a}_1 v \\ \vec{a}_2 v \\ \vdots \\ \vec{a}_m v \end{bmatrix}$$

So far

① $\vec{v}\,u$ = Scalar product

$\vec{v}$ ↑ row vector u ↖ col. vector

② $$A v = \begin{bmatrix} \vec{a}_1 v \\ \vec{a}_2 v \\ \vdots \\ \vec{a}_m v \end{bmatrix}$$

b/c $A = \begin{bmatrix} \vec{a}_1 \\ \vec{a}_2 \\ \vdots \\ \vec{a}_m \end{bmatrix}$

③ Also $A = [a_1 \ a_2 \cdots a_n]$

$$v = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$$

$$A v = [a_1 \ a_2 \cdots a_n] \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} = (a_1)(v_1) + (a_2)(v_2) + \cdots + (a_n)(v_n)$$

☆ linear combination of A's columns

$$A v = v_1 a_1 + v_2 a_2 + \cdots + v_n a_n$$

↑ ↑ ↑

$$\underset{2 \times 3}{\begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \end{bmatrix}} \underset{3 \times 1}{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}} = \begin{bmatrix} -1 \\ 13 \end{bmatrix}$$

$$[2 \ 1 \ 3] \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\underset{2\times 3}{\begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \end{bmatrix}} \underset{3\times 1}{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}} = 1\cdot\begin{bmatrix} 1 \\ 2 \end{bmatrix} + 2\begin{bmatrix} -1 \\ 1 \end{bmatrix} + 3\begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} -2 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 9 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} -1 \\ 13 \end{bmatrix}}$$

Matrix · Matrix?

$\underline{A} \cdot B$

$$\underset{k\times n}{B} = [\mathbf{b}_1 \ \mathbf{b}_2 \ \cdots \ \mathbf{b}_n] = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{2n} & & b_{2n} \\ \vdots & & & \\ b_{k1} & b_{kn} & \cdots & b_{kn} \\ \mathbf{b}_1 & & & \end{bmatrix}$$

$$\underset{m\times k}{\underline{A}} \cdot \underset{k\times n}{\underline{B}} = \textcircled{A}[\mathbf{b}_1 \ \mathbf{b}_2 \ \cdots \ \mathbf{b}_n]$$

$$= [\underline{A\mathbf{b}_1} \ \boxed{A\mathbf{b}_2} \ \cdots \ \underline{A\mathbf{b}_n}]$$

Note:

$$A\mathbf{b}_1 = \begin{bmatrix} \vec{a}_1 \mathbf{b}_1 \\ \vec{a}_2 \mathbf{b}_1 \\ \vdots \\ \vec{a}_m \mathbf{b}_1 \end{bmatrix}$$

So $\underset{m\times k}{A}\,\underset{k\times n}{B} = \begin{bmatrix} \boxed{\vec{a}_1 \mathbf{b}_1} & \boxed{\vec{a}_1 \mathbf{b}_2} & & \\ \boxed{\vec{a}_2 \mathbf{b}_1} & \boxed{\vec{a}_2 \mathbf{b}_2} & & \\ \vdots & \vdots & \ddots & \\ \boxed{\vec{a}_m \mathbf{b}_1} & \boxed{\vec{a}_m \mathbf{b}_2} & & \boxed{\vec{a}_m \mathbf{b}_n} \end{bmatrix} = C$

$m \times n$

$$C = \left[\vec{a}_i \mathbf{b}_j \right]$$