# Math 511

Due Tues 1.4 (1, 2, 3, 7, 9, 12, 14*, 15, 16*, 19)

$$\begin{bmatrix} -1 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \end{bmatrix} \qquad \begin{matrix} \vec{a}_1 \cdot v \\ \vec{a}_2 \cdot v \end{matrix}$$

2 X 3    3 X 1    2 X 1

$$(-1)\begin{bmatrix} -1 \\ 1 \end{bmatrix} + (2)\begin{bmatrix} 0 \\ 2 \end{bmatrix} + (3)\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 3 \\ -1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -3 & -1 & -3 \\ -2 & 1 & \boxed{3} \end{bmatrix}$$

2 X 2    2 X 3    2 X 3

**Facts:**

row vector · col vector = Scalar (scalar product)

col. vector · row vector = Matrix (outer product)

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 & 2 \\ -2 & 0 & 2 & 4 \\ -3 & 0 & 3 & 6 \end{bmatrix}$$

3 X 1    1 X 4

@ end of 1.3 we can ...

1. $A = B$
2. $2A$
3. $A + B$
4. $A - B$
5. $A\mathbf{x}$ — remember: row vector · col. vector = scalar
6. $AB$
7. $A^T$

Arithmetic

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## 1.4 Algebra

ex: College Algebra (Algebra on Reals)

ex: $\boxed{3x + x^2 = 4}$

Algebraic Expressions — b/c "=" Algebraic Equality

task: solve

??

$$3x + x^2 = 4$$

$$x^2 + 3x - 4 = 0$$

$$x^2 + 3x + \left(\tfrac{3}{2}\right)^2 - \left(\tfrac{3}{2}\right)^2 - 4 = 0$$

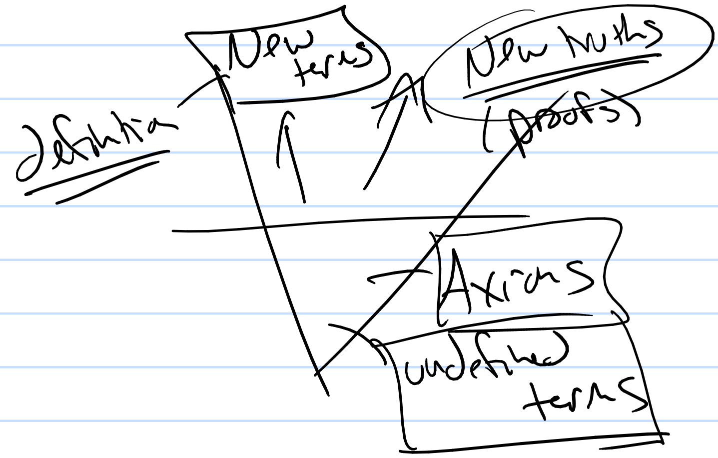
$$\left(x + \tfrac{3}{2}\right)^2 - \frac{25}{4} = 0$$

$$\left(\left(x + \tfrac{3}{2}\right) - \tfrac{5}{2}\right)\left(\left(x + \tfrac{3}{2}\right) + \tfrac{5}{2}\right) = 0$$

$$(x-1)(x+4) = 0$$

$x = 1$ $\quad$ $x = -4$

# Matrix Algebra




## Thu

① $A+B=B+A$

**Note:** $AB \neq BA$

easy example

$$\underset{2\times3}{A}\ \underset{3\times4}{B} = \underset{2\times4}{C}$$

$$\underset{3\times4}{B}\ \underset{2\times3}{A} \overset{?}{=} \times$$

$4 \neq 2$

Multiply does not commute

So Matrix Multiply has two versions.

$AB$

A mult. from left. (leftside multiply)

B multiplies from right side (right side multiply)

② $(A+B)+C = A+(B+C)$

③ $(AB)C = A(BC)$

④ $A(B+C) = AB + AC$

⑤ $(B+C)A = BA + CA$

⑥ $(\alpha\beta)A = \alpha(\beta A)$

⑦ $\alpha(AB) = (\alpha A)\cdot B = A\cdot(\alpha B)$

⑧ $(\alpha+\beta)A = \alpha A + \beta A$

⑨ $\boxed{\alpha(A+B)} = \boxed{\alpha A + \alpha B}$

$$A = [a_{ij}] = \begin{bmatrix} \vec{a}_1 \\ \vec{a}_2 \\ \vdots \\ \vec{a}_m \end{bmatrix} = [a_1 \; a_2 \; \cdots \; a_n]$$

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Two Special Matrices:

$$0 = [\text{all zeros}]$$

$$\underset{n\times n}{I} = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}$$ square matrix

$$I_{3\times3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

**Def**

integer powers of 0, 1, 2, 3, ...

$A_{n\times n}$ | $A^0 = I$
$A^{k+1} = A^k \cdot A$

ex $A^3 = A \cdot A \cdot A$

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**Note:** given an element and a binary operator we look for ① Identities

② Inverses.

**Identity:** $e \odot I = e$ "do nothing"

(operator symbol is $\odot$)

**Inverse:** $\exists$ some special element related to $e$ such that $e \odot e^{-1} = I$

inverse of $e$ under operator

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**Matrix Addition**

Additive identity

$A + \mathcal{O} = A$

$A + \boxed{(-1)A} = \mathcal{O}$

A's additive inverse

Multiplication Inverse

$$A \cdot I = A$$

identity

$$A \cdot (?) = I$$

call it $A^{-1}$

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$$AB + C = D$$
$$-C \qquad -C$$

$$AB = (D - C)$$

$$A \underbrace{\boxed{BB^{-1}}}_{I} = (D - C)B^{-1}$$

$$\boxed{A = (D - C)B^{-1}}$$