Math 511

Q's

1.3 #11 $A = [\vec{a}_1 \ \vec{a}_2 \ \vec{a}_3] = \begin{bmatrix} \vec{a}_1 \\ \vec{a}_2 \\ \vec{a}_3 \\ \vec{a}_4 \\ \vec{a}_5 \end{bmatrix}$

5×3 — rows, cols

if $\boxed{b = a_1 + a_2 = a_2 + a_3}$

and $Ax = b$ — matrix, col. vector, col. vector

Scratch

?, $Ax = \begin{bmatrix} \vec{a}_1 x \\ \vec{a}_2 x \\ \vec{a}_3 x \\ \vec{a}_4 x \\ \vec{a}_5 x \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{bmatrix} = b$

$Ax = [a_1 \ a_2 \ a_3] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$= x_1 a_1 + x_2 a_2 + x_3 a_3$

So $Ax = b$ can be thought of as...

$x_1 a_1 + x_2 a_2 + x_3 a_3 = b$

Solve: Find x_1, x_2, x_3 so that this equality is true.

Know: (told) $b = a_1 + a_2 = (1)a_1 + (1)a_2 + (0)a_3$

so $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ is a soln to $Ax = b$

(too)

$\mathbb{b} = a_1 + a_2 = a_2 + a_3$

$\hookrightarrow \mathbb{b} = a_1 + a_2 = (1)a_1 + (1)a_2 + (0)a_3$

$\mathbb{b} = a_2 + a_3 = (0)a_1 + (1)a_2 + (1)a_3$

so $\begin{bmatrix}1\\1\\0\end{bmatrix}$ and $\begin{bmatrix}0\\1\\1\end{bmatrix}$ solve $A\mathbf{x} = \mathbb{b}$

$\Downarrow$

blah blah blah $\boxed{\text{So}}$ $A\mathbf{x} = \mathbb{b}$ has an infinite number of solutions.

Note: if you write only what I wrote above ... (wrongly) need to write the spoken stuff.

(v) MLN

A is $m \times n$ and is underdetermined. All underdet systems have either zero or ∞ solutions. Quiz

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(vi) A is 5×3 is overdet. ($m > n$) All overdet systems can still have zero or 1 or ∞ solutions.

under det. 3 variable 1x3 or $\boxed{2 \times 3}$

Zero solutions

$$\boxed{\begin{aligned} x+y+z&=1 \\ x+y+z&=2 \end{aligned}}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 \end{array}\right]$$

$-r_1+r_2$
$= Nr_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

$0 \neq 1$!

$$\boxed{\begin{aligned} x+y+z&=3 \\ x-y+z&=1 \end{aligned}}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & -1 & 1 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} \boxed{1} & 1 & 1 & 3 \\ 0 & \boxed{1} & 0 & 1 \end{array}\right]$$

12 $A_{3\times 4}$

eqns vars.

$3 < 4$

under det.

$Ax = b$ soln $\begin{bmatrix} \vdots \end{bmatrix}$

$$\boxed{b = a_1 + a_2 + a_3 + a_4} = (1)a_1 + (1)a_2 + (1)a_3 + (1)a_4$$

$$A = [a_1 \; a_2 \; a_3 \; a_4] = \begin{bmatrix} \vec{a}_1 \\ \vec{a}_2 \\ \vec{a}_3 \end{bmatrix}$$

Matrix Algebra

Inverses / Identities

Matrix Addition

all zero matrix

$$A + O = A$$

Additive Identity

Additive Inverse

$$A + (-1)A = O$$

uniq. pairs

ex $\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ -3 & -4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Matrix Multiply

$$\underset{m \times n}{A} \cdot \underset{n \times n}{I} = \underset{m \times n}{A}$$

$\boxed{\text{Multiplicative Identity}}$ $\underset{3\times3}{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

ex $A \cdot I$

$\begin{bmatrix} a_{11} & a_{12} & a_{13} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} a_{11} & a_{12} & a_{13} \end{bmatrix} = A$

$\boxed{\text{Multiplicative Inv.}}$ call A^{-1}

① want $A^{-1}A = I$

$A A^{-1} = I$

says A must be square / $n \times n$

So for A^{-1} to exist ..

(1) A must be square.

(2) ???

Terms

Note from College Algebra ..

1st inverse is to undo an operation.

ex $3x = 4$

Mult. inv of 3 $\left(\frac{1}{3}\right) 3x = \left(\frac{1}{3}\right) 4$

$$x = 4/3$$

2nd what can not be undone?

$$0(x) = 0$$

$\boxed{\text{0 has no mult. inv.}}$ 0 · nothing = 1

or we can say mult. by zero (multiplicative annihilator) destroys the ability to find the unknown.

Matrices

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \overset{?}{=} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

terms

A^{-1} exists ① we call A invertable

② we also call A non-singular

A^{-1} does not exist ① we call A non-invertable

② we also call A singular.

goal (1) thm

$\boxed{A\mathbb{X} = \mathbb{O}}$ and A is not all zeros. and $\mathbb{X}$ is not all zeros

so A is "bad"

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to show inverses $AA^{-1} = A^{-1}A = I$

and A is $n \times n$

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① How to find A^{-1} if it exists

② How to know if A^{-1} exists at all

Tie to $\boxed{A\mathbb{X} = \mathbb{O}}$

homogeneous eqn

$$\left[\begin{array}{ccc|c} a_{11} & a_{12} & \dots & \mathbb{b} \end{array}\right]$$

row ops

$$\left[\begin{array}{cc|c} 1 & 0 & \text{ans} \\ 0 & 1 & \end{array}\right]$$

$A\mathbb{X} = \mathbb{b}$

$\cancel{\exists}\, EA\mathbb{X} = E\mathbb{b}$

$I\mathbb{X}$

$\mathbb{X} = \boxed{}$