Math 511

Due Monday 1.5 (8, 10 bdfh, 12ac, 15, 16*, 17*, 18*, 22*)

Q's 1.4 #12 A and B are inv when

$$AB = I$$
$$BA = I$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad A^{-1} = \frac{1}{d}\begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

$$d = a_{11}a_{22} - a_{21}a_{12}$$

$$= \begin{bmatrix} \dfrac{a_{22}}{a_{11}a_{22}-a_{21}a_{12}} & \dfrac{-a_{12}}{a_{11}a_{22}-a_{21}a_{12}} \\ \dfrac{-a_{21}}{a_{11}a_{22}-a_{21}a_{22}} & \dfrac{a_{11}}{a_{11}a_{22}-a_{21}a_{12}} \end{bmatrix}$$

Show $A^{-1}A = I$ Show $AA^{-1} = I$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \dfrac{a_{22}}{a_{11}a_{22}-a_{21}a_{12}} & \dfrac{-a_{12}}{a_{11}a_{22}-a_{21}a_{12}} \\ \dfrac{-a_{21}}{a_{11}a_{22}-a_{21}a_{22}} & \dfrac{a_{11}}{a_{11}a_{22}-a_{21}a_{12}} \end{bmatrix} = \cdots = I$$

or $A(2B) = 2[AB]$

$$A A^{-1} = \frac{1}{d}\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} a_{22} & -a_{21} \\ -a_{12} & a_{11} \end{bmatrix}$$

$$= \frac{1}{d}\begin{bmatrix} a_{11}a_{22} - a_{12}a_{21} & 0 \\ 0 & a_{11}a_{22} - a_{12}a_{21} \end{bmatrix}$$

$$= \frac{1}{d}\begin{bmatrix} d & 0 \\ 0 & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Note: $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ for any reals a_{ij}

$$A^{-1} = \frac{1}{\boxed{a_{11}a_{22} - a_{21}a_{22}}}\begin{bmatrix} a_{22} & -a_{21} \\ -a_{12} & a_{11} \end{bmatrix}$$

We know A^{-1} can exist .. how to find it?

$\boxed{\text{go back to systems of eqns}}$

system of eqn's [determined] A is $n \times n$

augmented matrix

$$[\, A \mid b \,]$$

Matrix eqn

$$A\mathbf{x} = \mathbf{b}$$

do a row op — step1 — $E_1 A\mathbf{x} = E_1 \mathbf{b}$

do a row op — step2 — $E_2E_1A\mathbf{x} = E_2E_1\mathbf{b}$

$$\underbrace{\boxed{E_n \cdots E_2E_1 A}}_{I}\mathbf{x} = \underbrace{\boxed{E_n \cdots E_2E_1}}_{A^{-1}}\mathbf{b}$$

$$A^{-1}A\mathbf{x} = A^{-1}\mathbf{b}$$

make it reduced row eschl. form

$$[\, I \mid \text{soln} \,]$$

$$I\mathbf{x} = A^{-1}\mathbf{b}$$

Fact #1

$$AB = C$$

$$B = A^{-1}C$$

Soln

$$[\, A \mid C \,]$$

row ops

$$[\, I \mid A^{-1}C \,]$$

Fact #2

$$[A \mid I] \xrightarrow{\text{row ops}} [I \mid A^{-1}I]$$

$$[A \mid I] \xrightarrow{\text{row ops}} [I \mid A^{-1}]$$

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ex

$$\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ -1 & 2 & 0 & 1 \end{array}\right] \underset{r_1+r_2}{=} \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & 5 & 1 & 1 \end{array}\right]$$

$$\tfrac{1}{5} r_2 \to \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & 1 & 1/5 & 1/5 \end{array}\right] \xrightarrow{r_1 - 3r_2} \left[\begin{array}{cc|cc} 1 & 0 & 2/5 & -3/5 \\ 0 & 1 & 1/5 & 1/5 \end{array}\right]$$

$$\begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 2/5 & -3/5 \\ 1/5 & 1/5 \end{bmatrix}$$

Now: Find matrices, E, that do exactly what row ops do.

Warning for textbook notation (vs. Me)

① If you do row op 1 then row op 2 then ..

$$E_n \cdots E_5 \overset{T2}{E_4} \overset{T1}{E_3} \overset{T1}{E_2} \overset{T3}{E_1} A$$

4th 3rd 2nd 1st row performed

② 3 types of row ops

type #1	$E_{type1} = E^{T1}$	E_1
type #2	$E_{type2} = E^{T2}$	E_2
type #3	$E_{type3} = E^{T3}$	E_3
	Mark notation	Books

$\boxed{\text{Type 1}}$ E^{T_1} row swap.

take I and swap row i and row j

then this is $E^{T_1} = \left[\begin{array}{l} I \text{ but you} \\ \text{swap rows } i,j \end{array}\right]$

ex

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix} = \begin{bmatrix} 9 & 10 & 11 & 12 \\ 5 & 6 & 7 & 8 \\ 1 & 2 & 3 & 4 \\ 13 & 14 & 15 & 16 \end{bmatrix}$$

also: $(E^{T_1})^{-1}$ is just the same E^{T_1}

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type 2 $\quad m \cdot row_i = \text{New } r_i$

$$E^{T_2} = \left[I, \text{ but you place } m \text{ in the } row_i \text{ diag. spot} \right]$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/2 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 1 \\ 4 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 1 \\ 2 & 1 & 1/2 \end{bmatrix}$$

$$(E^{T_2})^{-1} = \left[I, \text{ but you place } \tfrac{1}{m} \text{ in the } row_i \text{ diag. spot} \right]$$

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ex $\quad 5 \times 5 \quad$ want to mult. row 2 by π

$$E^{T_2} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$\boxed{\text{Type 3}}$ row $\underline{i}$ + M · row $\underline{j}$ = New row i

$$E^{T_3} = \left[I, \text{ but you put an } M \text{ in } a_{ij} \text{ spot} \right]$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -1/4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & 3 \\ 1/4 & 4 & 3 \\ 2 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 9/2 & 9/4 \\ 2 & 1 & 1 \end{bmatrix}$$

$r_2 + (-1/4) r_1 = N\, r_{\underline{2}}$

$$\left(E^{T_3}\right)^{-1} = \left[I, \text{ put } -M \text{ in } a_{ij} \text{ spot} \right]$$