Math 511

<u>Due Wed</u> 1.5 (b/c I forgot no classes on Monday)

1.6 (1, 2*, 4a, 5a, 8, 13*, 14)

No Quiz

$$\begin{aligned} 2x_1 - x_2 + x_3 &= 3 \\ x_1 + 2x_2 - x_3 &= 0 \\ 3x_1 + x_2 + x_3 &= 8 \end{aligned}$$

$A\mathbb{x} = \mathbb{b}$

$$\left[\begin{array}{ccc|c} 2 & -1 & 1 & 3 \\ 1 & 2 & -1 & 0 \\ 3 & 1 & 1 & 8 \end{array}\right]$$

$$E_4^{T_3} E_3^{T_3} E_2^{T_3} E_1^{T_1} \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 8 \end{bmatrix}$$

E_1: $r_1 \leftrightarrow r_2$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 2 & -1 & 1 & 3 \\ 3 & 1 & 1 & 8 \end{array}\right] \begin{array}{l} \leftarrow \\ Nr_2 = r_2 + (-2) r_1 \quad E_2^{T_3} \\ Nr_3 = r_3 + (-3) r_1 \quad E_3^{T_3} \end{array}$$

$$E_1^{T_1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & -5 & 3 & 3 \\ 0 & -5 & 4 & 8 \end{array}\right] \quad Nr_3 = r_3 + (-1) r_2 \quad E_4^{T_3}$$

$$E_2^{T_3} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad E_3^{T_3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$$

$$E_4^{T_3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & -5 & 3 & 3 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

Pause

$$\left[\begin{array}{ccc|c} 2 & -1 & 1 & 3 \\ 1 & 2 & -1 & 0 \\ 3 & 1 & 1 & 8 \end{array}\right] \qquad \left(E_4^{T_3} E_3^{T_3} E_2^{T_3} E_1^{T_1} \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 3 & 1 & 1 \end{bmatrix}\right) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 8 \end{bmatrix}$$

E_1

$r_1 \leftrightarrow r_2$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 2 & -1 & 1 & 3 \\ 3 & 1 & 1 & 8 \end{array}\right] \quad \begin{array}{l} Nr_2 = \boxed{r_2 + (-2) r_1} \quad E_2^{T_3} \\ Nr_3 = r_3 + (-3) r_1 \quad E_3^{T_3} \end{array}$$

$$E_1^{T_1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & -5 & 3 & 3 \\ 0 & -5 & 4 & 8 \end{array}\right] \quad Nr_3 = r_3 + (-1) r_2 \quad E_4^{T_3}$$

$$E_2^{T_3} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad E_3^{T_3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & -5 & 3 & 3 \\ 0 & 0 & 1 & 5 \end{array}\right] \qquad E_4^{T_3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}}_{E_4} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}}_{E_3} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{E_2} \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{E_1} \underbrace{\begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 3 & 1 & 1 \end{bmatrix}}_{A} = \underbrace{\begin{bmatrix} 1 & 2 & -1 \\ 0 & -5 & 3 \\ 0 & 0 & 1 \end{bmatrix}}_{U}$$

$$E_3 E_2 E_1 A = E_4^{-1} U$$

$$A = \left(E_1^{-1} E_2^{-1} E_3^{-1} E_4^{-1}\right) U$$

$\boxed{\text{Fact}}$ if you $\boxed{\text{only}}$ do type 3 matrices

If you use $\boxed{\text{only}}$ type 3 matrices then

$\boxed{\text{Fact}}$

$$A = \underbrace{(E_1^{-1} E_2^{-1} \cdots E_k^{-1})}_{\text{all are lower triangular and if you multiply them together it is lower triangular}} U$$

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So $\boxed{A = L \cdot U}$ L,U factorization

ex

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1/3 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A = U$$

$$A = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1/3 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}} U$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1/3 & 2 & 1 \end{bmatrix} U$$

$\boxed{\text{Unpause}}$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & -5 & 3 & 3 \\ 0 & 0 & \boxed{1} & 5 \end{array}\right]$$

$Nr_1 = r_1 + (1) r_3$ $\quad E_6 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$Nr_2 = r_2 + (-3) r_3$ $\quad E_5 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 5 \\ 0 & -5 & 0 & -12 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$Nr_2 = -\frac{1}{5} r_2$ $\quad E_7 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1/5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 5 \\ 0 & -5 & 0 & -12 \\ 0 & 0 & 1 & 5 \end{array}\right] \quad N r_2 = -\tfrac{1}{5} r_2 \qquad E_7 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1/5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 5 \\ 0 & 1 & 0 & 12/5 \\ 0 & 0 & 1 & 5 \end{array}\right] \quad N r_1 = r_1 + (-2) r_2 \qquad E_8 = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1/5 \\ 0 & 1 & 0 & 12/5 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$$(\underbrace{E_8 E_7 E_6 E_5 E_4 E_3 E_2 E_1}_{= A^{-1}}) A = I$$

$$[A \mid I] \rightsquigarrow [I \mid A^{-1}]$$

What should we be able to do?

(1) $[A \mid b]$

$\rightsquigarrow$

$[U \mid \ ]$

$E_1 = ?$

$E_2 = ?$

write matrix version of row ops.

(2) $A = \boxed{L}\,\boxed{U}$ (remember $\boxed{\text{only}}$ use type 3)

(3) $[A \mid I]$

$\rightsquigarrow$

$[I \mid A^{-1}]$

find

$\boxed{\text{or}}$ want $A^{-1}C$?

$[A \mid C]$

$\rightsquigarrow$

$[I \mid A^{-1}C]$

1.6 partitioned matrix

$$\left[\begin{array}{cc|c} 1 & 2 & 3 \\ \hline 4 & 5 & 6 \\ 7 & 8 & 9 \end{array}\right] = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

$$\left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right] \left[\begin{array}{c|c|c} B_{11} & B_{12} & B_{13} \\ \hline B_{21} & B_{22} & B_{23} \end{array}\right]$$

2×2 $\quad$ 2×3

$$= \left[\begin{array}{c|c|c} A_{11}B_{11} + A_{12}B_{21} & & \\ \hline & & \end{array}\right]$$