Math 511

Q's 1.6 #8 $A_{m\times n}$, $X_{n\times r}$, $B_{m\times r}$

fact $AX = B$ iff $A\mathbf{x}_j = \mathbf{b}_j$ $j = 1, 2, \ldots, r$

to show left side iff right side is true.

logically equiv.

Note

tech #1 left side same as step 1 same as step 2

... same as right side

tech #2 left side implies right side

and

right side implies left side

Fact $AX = B$ iff $A\mathbf{x}_j = \mathbf{b}_j$ $j = 1, 2, \ldots, r$

A is $m\times n$

X is $n\times r$

B is $m\times r$

$X = [\mathbf{x}_1\ \mathbf{x}_2\ \ldots\ \mathbf{x}_r]$

$B = [\mathbf{b}_1\ \mathbf{b}_2\ \ldots\ \mathbf{b}_r]$

Scratch $AX = B$

iff $A[\mathbf{x}_1\ \mathbf{x}_2 \cdots \mathbf{x}_r] = [\mathbf{b}_1\ \mathbf{b}_2 \cdots \mathbf{b}_r]$

iff $[A\mathbf{x}_1\ A\mathbf{x}_2 \cdots A\mathbf{x}_r] = [\mathbf{b}_1\ \mathbf{b}_2 \cdots \mathbf{b}_r]$

iff when cols. equal

$A\mathbf{x}_1 = \mathbf{b}_1$ and $A\mathbf{x}_2 = \mathbf{b}_2$ and $\cdots$ $A\mathbf{x}_r = \mathbf{b}_r$

short hand notation $A\mathbf{x}_j = \mathbf{b}_j \quad j = 1, 2, 3, \ldots, r$

1.5 12a Solve -- $X = ?$

$A = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$

$B = \begin{bmatrix} 6 & 2 \\ 2 & 4 \end{bmatrix}$

$C = \begin{bmatrix} 4 & -2 \\ -6 & 3 \end{bmatrix}$

$AX + B = C$

$AX = [C - B]$

$A^{-1}AX = A^{-1}[C - B]$

$X = A^{-1}[C - B]$

$\left[\begin{array}{cc|cc} 5 & 3 & 1 & 0 \\ 3 & 2 & 0 & 1 \end{array}\right]$

$\left[\begin{array}{cc|cc} 1 & 3/5 & 1/5 & 0 \\ 3 & 2 & 0 & 1 \end{array}\right]$

$\left[\begin{array}{cc|cc} 5 & 3 & 1 & 0 \\ 0 & 1 & -3 & 5 \end{array}\right]$

$\left[\begin{array}{cc|cc} 5 & 0 & 10 & -15 \\ 0 & 1 & -3 & 5 \end{array}\right]$

so

$A^{-1} = \begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix}$

$X = ?$

or $\Rightarrow$ $X = A^{-1}(C-B)$

$A = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$

$B = \begin{bmatrix} 6 & 2 \\ 2 & 4 \end{bmatrix}$

$C = \begin{bmatrix} 4 & -2 \\ -6 & 3 \end{bmatrix}$

$X = A^{-1}\begin{bmatrix} -2 & -4 \\ -8 & -1 \end{bmatrix}$

$$\left[\begin{array}{cc|cc} 5 & 3 & -2 & -4 \\ 3 & 2 & -8 & -1 \end{array}\right]$$

$\Downarrow$ row ops

$$\left[\begin{array}{cc|c} 1 & 0 & A^{-1}\begin{bmatrix} -2 & -4 \\ -8 & -1 \end{bmatrix} \\ 0 & 1 & \end{array}\right]$$

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ex

$XA + A = C$

$XA + IA = C$

$(X+I)A = C$

$X + I = CA^{-1}$

$X = \underline{CA^{-1} - I}$

ch 1 — Important thm's

1.5.2 — Logically Equiv. Statements

1. A is non-singular
2. A^{-1} exists
3. $A\mathbb{x} = \mathbb{0}$ has only trivial soln. $(\mathbb{x} = 0)$
4. A is row equiv. to I

$$\underbrace{E_4 \cdots E_3 E_2 E_1}_{\text{row ops}} A = I$$

1. A is singular
2. A^{-1} does not exist
3. $A\mathbb{x} = \mathbb{0}$ has a non-trivial soln
4. A is not row equiv to I.

Corollary

$$A\mathbb{x} = \mathbb{b} \quad (\overbrace{n \text{ eqn's}, n \text{ unknowns}}^{\text{determined system}})$$

$\mathbb{x}$ is a uniq. soln iff A is non-singular.

(⇐) ⟶

if $\mathbb{x}$ is a uniq. soln and A is singular

$A\mathbb{x} = \mathbb{b}$ — to homog.

have a non-trivial soln $\hat{\mathbb{x}}$

$$A\hat{\mathbb{x}} = \mathbb{0}$$

consider $A(\mathbb{x} + \hat{\mathbb{x}}) = A\mathbb{x} + A\hat{\mathbb{x}} = A\mathbb{x} + 0 = \mathbb{b}$

new soln !

$\boxed{\text{Ch 2 Determinants}}$ to determine if A^{-1} exists

A is $n \times n$, $n = 1, 2, 3, \ldots$

A is 1×1 $\quad A = [a_{11}]$ $\quad A^{-1} = \left[\frac{1}{a_{11}}\right]$ $\quad$ if $a_{11} \boxed{\neq 0}$ A^{-1} exists

A is 2×2 $\quad A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ $\quad A^{-1} = \frac{1}{a_{11}a_{22} - a_{12}a_{21}} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$

A is 3×3 $\quad A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ $\quad$ if $a_{11}a_{22} - a_{12}a_{21} \boxed{\neq 0}$ A^{-1} exists

$\underline{\underline{A \text{ is } 3\times3}}$ $\quad [A \mid I] \sim [I \mid A^{-1}]$

$$A^{-1} = \frac{1}{d}\left[\;\Sigma\;\right]$$

$$d = a_{11}a_{22}a_{33} - a_{11}a_{32}a_{23} - a_{12}a_{21}a_{33} + a_{12}a_{31}a_{23} + a_{13}a_{21}a_{32} - a_{13}a_{32}a_{22}$$

$\neq 0$ says A^{-1} exists

$$= a_{11}(a_{22}a_{33} - a_{32}a_{23})$$

old thing

Determinants By Cofactor Expansion

$$A_{n\times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & & & \\ a_{n1} & - & - & a_{nn} \end{bmatrix}$$

(Next class)