Math 511

Q's | 2.2 #12

$\det(A)$ → tech #1 – cofactor expansion
→ tech #2 – by elimination (row equiv. matrices)

ex 3

$$\begin{vmatrix} 1 & -2 & 3 \\ 2 & 0 & 1 \\ 3 & -1 & 2 \end{vmatrix} = -(-2)\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} + 0 - (-1)\begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix}$$

cofactor expansion

$$= 2(4-3) + (1-6)$$
$$= 2 + -5 = -3$$

b/c $\det(A) \neq 0$ A is invertible
or A^{-1} exists
or A is non-singular
or A is row equiv. to I
or $A\mathbf{x} = 0$ has only trivial soln.

by elimination

$$\begin{vmatrix} 1 & -2 & 3 \\ \underline{2} & 0 & 1 \\ \underline{3} & -1 & 2 \end{vmatrix}$$

gauss elim

$\det(E_n \cdots E_2 E_1 A) = \det(U)$
or
$$\det(A) \equiv \frac{\det(U)}{\det(E_1)\det(E_2)\cdots\det(E_n)}$$

used E^{T_3}

$$= \begin{vmatrix} 1 & -2 & 3 \\ 0 & 4 & -5 \\ 0 & 5 & -7 \end{vmatrix}$$

New $r_3 = r_3 + -\frac{5}{4} r_2$

$$= \begin{vmatrix} 1 & -2 & 3 \\ 0 & 4 & -5 \\ 0 & 0 & -3/4 \end{vmatrix} = \boxed{-3}$$

$-7 + -\frac{5}{4}(-5)$

$-7 + \frac{25}{4}$

3X3 Vandermonde Matrix

#12

$$V = \begin{bmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \end{bmatrix}$$

a) $\det(V)$

$$\begin{vmatrix} 1 & x_1 & x_1^2 \\ \boxed{1} & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \end{vmatrix}$$

used E^{I3}

$$= \begin{vmatrix} 1 & x_1 & x_1^2 \\ 0 & x_2 - x_1 & x_2^2 - x_1^2 \\ 0 & x_3 - x_1 & x_3^2 - x_1^2 \end{vmatrix}$$

$$= 1 \cdot \begin{vmatrix} x_2 - x_1 & x_2^2 - x_1^2 \\ x_3 - x_1 & x_3^2 - x_1^2 \end{vmatrix} - 0 + 0$$

$$= (x_2 - x_1)(x_3^2 - x_1^2) - (x_3 - x_1)(x_2^2 - x_1^2)$$

$$= \underline{(x_2 - x_1)}\,\underline{(x_3 - x_1)}(x_3 + x_1) - \underline{(x_3 - x_1)}\,\underline{(x_2 - x_1)}(x_2 + x_1)$$

$$= (x_2 - x_1)(x_3 - x_1)\left[(x_3 + x_1) - (x_2 + x_1)\right]$$

$$= \boxed{(x_2 - x_1)(x_3 - x_1)(x_3 - x_2)}$$

Data fitting

Points: $(x_1, y_1), (x_2, y_2) \ldots$

fit it to a quadratic?

(Find a, b, c)

$$y = a + bx + cx^2$$

goal:

$$y_1 = a + bx_1 + cx_1^2$$
$$y_2 = a + bx_2 + cx_2^2$$
$$y_3 = a + bx_3 + cx_3^2$$
$$y_4 = a + bx_4 + cx_4^2$$
$$\vdots$$

$$(\star) \quad \begin{bmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \\ 1 & x_4 & x_4^2 \\ & \vdots & \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ \vdots \end{bmatrix}$$

Goal!

$$V\mathbf{c} = \mathbf{y}$$

$$\mathbf{c} = V^{-1}\mathbf{y}$$

2.3 Adjoint of a Matrix

cofactor $A_{ij} = (-1)^{i+j} \left| M_{ij} \right|$

with M_{ij} is the Minor matrix where A has row i and col j removed.

ex
$$\begin{vmatrix} 1 & 2 & 3 \\ 1 & 0 & 4 \\ 1 & -1 & 2 \end{vmatrix}$$

$a_{23} = 4$

$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix}$

$= - \begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix}$

$$\boxed{\operatorname{adj} A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T}$$

Idea:

$$A \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T = A \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$$= \begin{bmatrix} \vec{a_1} \\ \vec{a_2} \\ \vec{a_3} \end{bmatrix} \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} \det(A) & 0 & 0 \\ 0 & \det(A) & 0 \\ 0 & 0 & \det(A) \end{bmatrix}$$

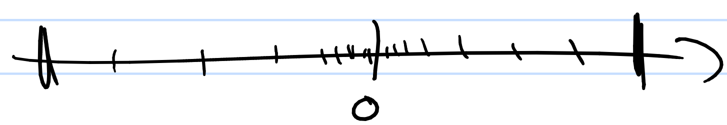
so $(A)(\text{adj } A) = \det(A)\, I$

$$\frac{1}{\det(A)}(A)(\text{adj } A) = I$$

$$(A)\left(\frac{1}{\det(A)}\text{adj } A\right) = I$$

so $\boxed{A^{-1} = \frac{1}{\det(A)}(\text{adj } A)}$

$1 \quad x_1 \quad x_1^2 \quad x_1^3 \quad x_1^4$



$$(x_1 \cdot x_2 \cdots x_n)^{1/n}$$

$$= \exp\left(\frac{\ln x_1 + \ln x_2 + \cdots + \ln x_n}{n}\right)$$

$$\underbrace{\underset{4\times 37}{V^T} \quad \underset{37\times 4}{V}}\alpha = y \qquad (V^T V)\alpha = V^T y$$

$$\alpha = (V^T V)^{-1} V^T y$$