Math 511

Q5

2.1 (3)

2.1 (cofactor expansion)

(g)
$$\begin{vmatrix} 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 6 & 2 & 0 \\ 1 & 1 & -2 & 3 \end{vmatrix}$$

$$= 0 + 1 \begin{vmatrix} 2 & 0 & 1 \\ 1 & 2 & 0 \\ 1 & -2 & 3 \end{vmatrix} - 0 + 0 = \begin{vmatrix} 2 & 0 & 1 \\ 1 & 2 & 0 \\ 1 & -2 & 3 \end{vmatrix}$$

$$= 2 \begin{vmatrix} 2 & 0 \\ -2 & 3 \end{vmatrix} - 0 + 1 \begin{vmatrix} 1 & 2 \\ 1 & -2 \end{vmatrix}$$

$$= 2(6+0) + (-2-2) = 12 - 4 = 8$$

$$\det(A) = 8$$

2.2 (by elimination)

$EA = B$

$\det(E)\det(A) = \det(B)$

$$\det(A) = \frac{1}{\det(E)} \det(B)$$

(g)
$$\begin{vmatrix} 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 6 & 2 & 0 \\ 1 & 1 & -2 & 3 \end{vmatrix}$$

$$= - \begin{vmatrix} 1 & 1 & -2 & 3 \\ 0 & 1 & 0 & 0 \\ 1 & 6 & 2 & 0 \\ 2 & 0 & 0 & 1 \end{vmatrix} = - \begin{vmatrix} 1 & 1 & -2 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 5 & 4 & -3 \\ 0 & -2 & 4 & -5 \end{vmatrix} = - \begin{vmatrix} 1 & 1 & -2 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & -3 \\ 0 & 0 & 0 & -2 \end{vmatrix}$$

$$= -(1 \cdot 1 \cdot 4 \cdot -2) = -(-8) = 8$$

Notation:

$$|A| = -1\begin{vmatrix} 1 & 1 & -2 & 3 \\ 0 & 1 & 0 & 0 \\ 1 & 6 & 2 & 0 \\ 2 & 0 & 0 & 1 \end{vmatrix} = -\begin{vmatrix} 1 & 1 & -2 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 5 & 4 & -3 \\ 0 & -2 & 4 & -5 \end{vmatrix} = 8$$

row equiv.

(B)

$$\begin{bmatrix} 1 & 1 & -2 & 3 \\ 0 & 1 & 0 & 0 \\ 1 & 6 & 2 & 0 \\ 2 & 0 & 0 & 1 \end{bmatrix} \xrightarrow[\text{equiv}]{\text{row}} \begin{bmatrix} 1 & 1 & -2 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 5 & 4 & -3 \\ 0 & -2 & 4 & -5 \end{bmatrix}$$

$$[A \mid b] \xrightarrow[\text{what you did}]{\text{s.e.y}} [I \mid A^{-1}b]$$

2.2.3 in lecture...

$$\det(EA) = \det(E)\det(A)$$

Thm 2.2.3

$$\det(AB) = \det(A)\det(B)$$

2.2 #7

$$\det(AB) = \det(A)\det(B)$$

$$\det(A) = 4 \quad \det(B) = 5 \quad \rightarrow = 20$$

$\det(3A)$ $\qquad$ $\det(AB) = \det(A)\det(B)$

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 3a_{11} & 3a_{12} & 3a_{13} \\ 3a_{21} & 3a_{22} & 3a_{23} \\ 3a_{31} & 3a_{32} & 3a_{33} \end{bmatrix}$$

$$\det(3A) = \det((3I)A)$$

Exam 1 $\qquad$ 12 probs @ 10pts

110pts = 100%

Study tips:

1. Do not cram!
2. sleep well on these!

Friday 17th $\qquad$ 1:30 pm $\qquad$ in JB 336

1.1 (2probs) $\qquad$ 1. Solve a system of eqn's without matrices.

2. Solve w/ aug. matrix (gauss-elim.) w/ back solve

1.2 (1 prob)

① Word problem → Solve by aug. Matrix
of Gauss-Jordan elimination

traffic or circuits

1.3 (2 probs) Arithmetic

①
② } Do lots of Arithmetic !

1.4 (1 prob)

① Do Algebra !

(assume inverses exist)

$X = ?$

$X - XB = A$

$X(I - B) = A$

$\boxed{X = A(I-B)^{-1}}$

1.5 (2 probs)

① LU Factorization of E^{Type}

② Find A^{-1}

$[A \mid I]$

$\Downarrow$

$[I \mid A^{-1}]$

1.6 (0 probs)

Ch 2   2.1 – 2.3   (4 probs)

Know Th$^{=}$ 1.5.2 w/ det(A) = 0 or det(A) ≠ 0

① Find det(A)

② Meaning of (det)'s (1x Vandermond last class)
  ^ find.

③ Meaning

④ Understanding row ops, row equiv, det and det of products.