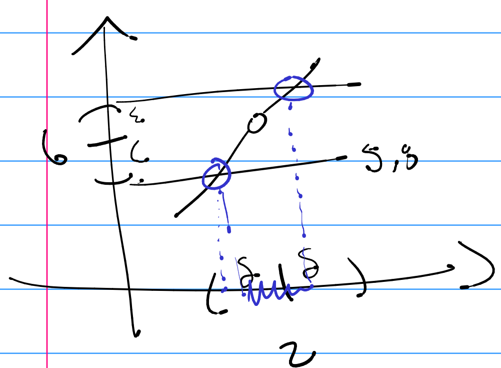
Math 242

~~Q's~~ 1.7 #7 $\lim_{x\to 2} (x^3 - 3x + 4) = 2^3 - 3(2) + 4$

$= 6$

ε-δ

$\forall \varepsilon > 0 \ \exists \delta > 0$ (if $0 < |x-2| < \delta$, then $|(x^3-3x+4) - 6| < \varepsilon$)



1.8 Continuity

Def f is continous @ $x = c$ if

$\boxed{\lim_{x\to c} f(x) = f(c)}$

(1) $\lim_{x\to c} f(x)$ exist

(2) $f(c)$ exist

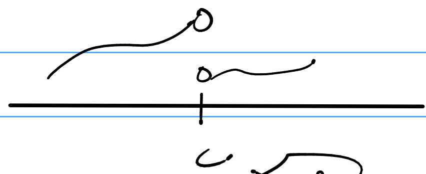
(3) equal $\lim_{x\to c} f(x) = f(c)$

If not $f(x)$ is discontinous @ $x = c$.

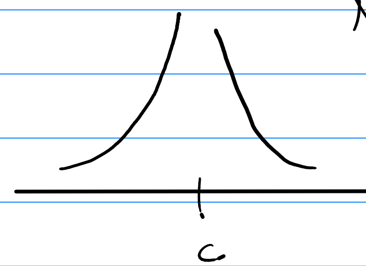
Discontinuous

① $\lim_{x \to c} f(x)$ does not exist

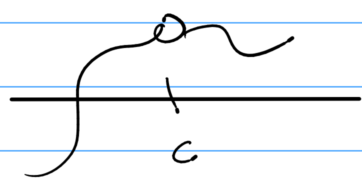
$\lim_{x \to c} f(x) = +\infty$ d.n.e

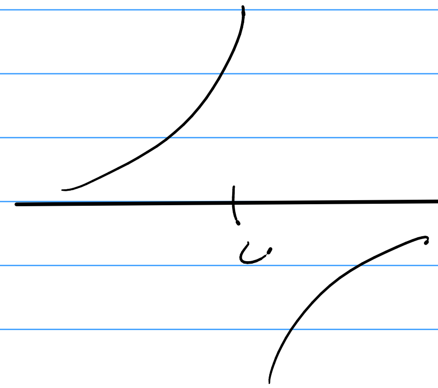


d.n.e

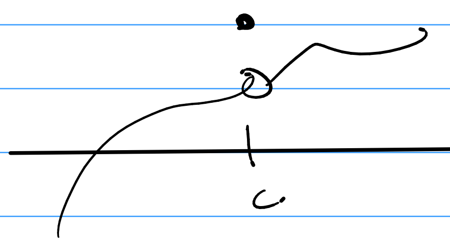


② $f(c)$ d.n.e





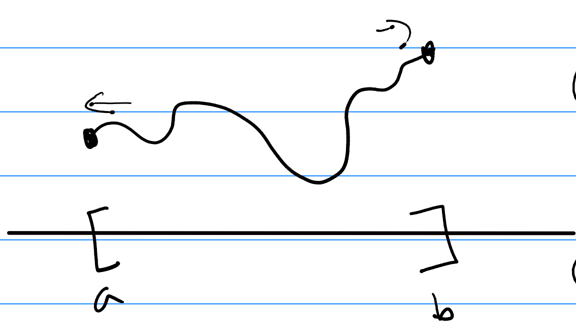
③ $\lim_{x \to c} f(x) \neq f(c)$



---

above is continuity @ a single point.

But functions have domains (intervals)



① $f(x)$ is cont from the right when $\lim_{x \to a^+} f(x) = f(a)$

② $f(x)$ is cont. from the left when $\lim_{x \to b^-} f(x) = f(b)$

$f(x)$ is cont on Interval (a,b) if
$\lim_{x\to c} f(x) = f(c)$ for all c in (a,b).

---

all 3 above say f is cont. on $[a,b]$.

Properties of Cont. functions

Thm if $f(x)$ and $g(x)$ are cont @ $x=a$
then

1. $(f \pm g)(x)$ is cont. @ $x=a$
2. $(fg)(x)$ is cont @ $x=a$
3. $\left(\frac{f}{g}\right)(x)$ is cont @ $x=a$
(if $g(a) \neq 0$)

So
1. $p(x)$ a polynomial is continuous on all real numbers. $\mathbb{R} = (-\infty, \infty)$
2. $r(x) = \frac{p(x)}{q(x)}$ a rational is continuous on all reals except where $q(x) = 0$

$\mathbb{R} - \{x \mid q(x) = 0\} = \{x \mid q(x) \neq 0\}$

So .. poly, rationals are cont. on their domains.

Thm: $f(x)$ is cont on its domain if

1. $f(x)$ is a polynomial
2. $f(x)$ is a rational
3. $f(x)$ is a root function
4. $f(x)$ is a trig function

Use: 1st

Cont. is $\lim_{x \to c} f(x) = f(c)$

Ex $\lim_{x \to 1} \left(\frac{x^2-1}{x+1} + \frac{1}{x} - \sin(x) + \tan(x) \right)$

$= \frac{1^2-1}{1+1} + \frac{1}{1} - \sin(1) + \tan(1) = \tan(1) - \sin(1) + 1$

Composition $(f \circ g)(x) = f(g(x))$

Thm $\lim_{x \to a} g(x) = b$ and f is cont @ b.

then $\lim_{x \to a} (f \circ g)(x) = \lim_{x \to a} f(g(x)) = f(b) = f\left(\lim_{x \to a} g(x)\right)$

Thm If g is cont @ a and f is cont @ $g(a)$ then

$(f \circ g)(x) = f(g(x))$ is cont @ a.

"a continuous function of a continuous function is continuous" (on its domain)

---

exs Intervals of continuity are really ...

"what is the (natural) domain?"

ex

$$f(x) = \frac{3 - \tan(x)}{\sqrt{x}}$$

3: cont everywhere

$\tan(x)$: cont $x \neq \frac{\pi}{2} \pm n\pi$

$\sqrt{x}$: cont on $x > 0$

Cont on $x > 0$ and $x \neq \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

ex $F(x) = \sin(\cos(\sin x))$ Composition of 3 cont. function $\to$ $\boxed{\text{cont}}$ on $\mathbb{R}$

ex $f(x) = \sqrt{1 + \frac{1}{x}}$ is cont. on $\boxed{1 + \frac{1}{x} \geq 0}$

$$1 + \frac{1}{x} \geq 0 \rightarrow \frac{x+1}{x} \geq 0$$

$$(-\infty, -1] \cup (0, +\infty)$$

$$\frac{x+1}{x} \rightarrow$$

+	−	+
	-1	0

(ex) $f(x) = \sqrt{1 + 1/x}$

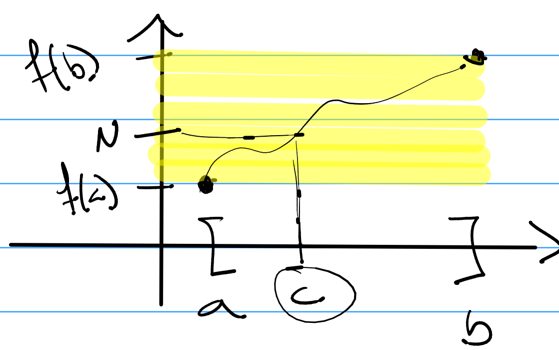
$$\lim_{x \to 2} \sqrt{1 + 1/x} = \sqrt{1 + 1/2}$$

So 1^{st} use was to easily do limits

2nd

Q does knowing something about the outside help us to know anything about the inside?

Intermediate Value theorem



Know: ① $f(x)$ is cont on $[a,b]$
② $f(a) \neq f(b)$

N is any number between $f(a)$ and $f(b)$ then there is some $x = c$ in (a,b) such that $f(c) = N$.

Using the intermediate value thm

ex) $\sqrt{2} = ?$

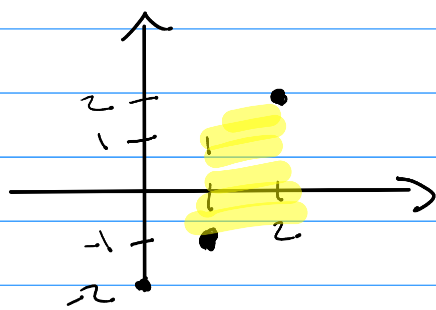
pos. root of

Solve $x \cdot x = 2$

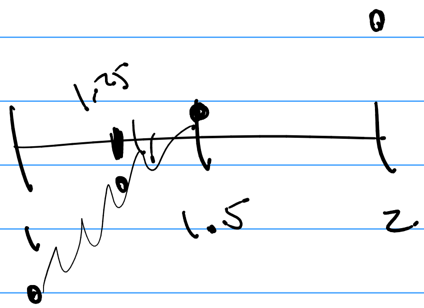
$x^2 = 2$

$x^2 - 2 = 0$

$f(x) = x^2 - 2$



b/c x^2-2 is cont. then $x^2-2=0$ between $x=1$ and $x=2$.



$(1.5)^2 - 2 = 2.25 - 2 = 0.25$