Math 242

Q's/ $\lim_{x\to 2} 3x-1 = 3\cdot 2-1 = 5$

$\forall \varepsilon>0\ \exists \delta>0$ (if $0<|x-2|<\delta$, then $|(3x-1)-5|<\varepsilon$)

Scratch:

ex $\lim_{x\to 2} x^2=4$

start

$\forall \varepsilon>0\ \exists \delta>0$ (if $0<\boxed{|x-2|<\delta}$, then $|x^2-4|<\varepsilon$)

Scratch: $|x^2-4|<\varepsilon \to |(x+2)(x-2)|<\varepsilon$

$\to |x+2|\boxed{|x-2|<\varepsilon}$

Know x is "close to 2"; make sure $x\in[1,3]$

$\delta=1$ at largest

+ $|x+2|$ is between $[3,5]$

ok $|x+2||x-2| < \boxed{5|x-2| < \varepsilon}$

Use this

$|x-2|<\varepsilon/5$

Pf $0<|x-2|<\delta$ let $\delta=\varepsilon/5$ or 1 (whichever is smaller)

so $|x-2|<\varepsilon/5 \to 5|x-2|<\varepsilon$

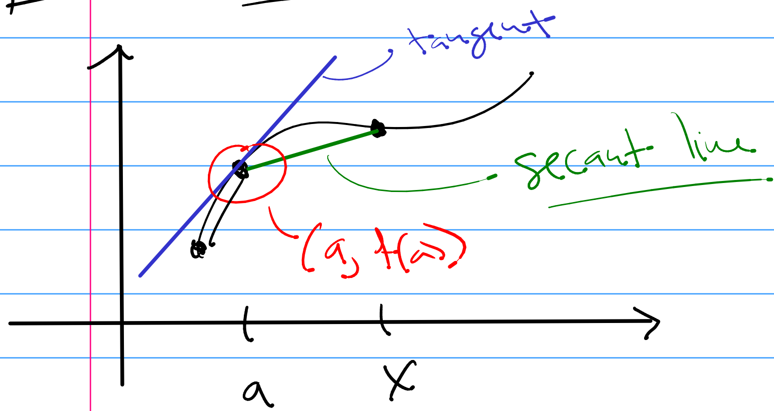
$\to \varepsilon > 5|x-2| > |x+2||x-2| = |x^2-4|$

b/c $\delta=\min(1,\varepsilon/5)$ so $3<|x+2|<5$ at worst

New math: $\lim_{x \to a} f(x) = L$

$\forall \varepsilon > 0\ \exists \delta > 0 \left(\text{if } 0 < |x - a| < \delta, \text{ then } |f(x) - L| < \varepsilon \right)$

Ch 2 **Derivatives**



line: defined by two pieces of info
① point
② slope

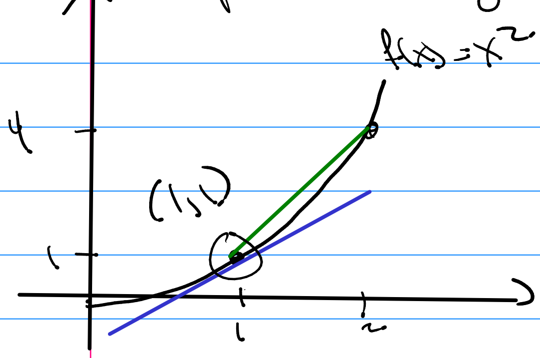
slope of secant is $\qquad M_{\text{secant}} = \dfrac{f(x) - f(a)}{x - a}$

slope of tangent is $\qquad M_{\text{tangent}} = \lim_{x \to a} \dfrac{f(x) - f(a)}{x - a}$

Ex $\boxed{f(x) = x^2}$

slope of secant between $x = 1$ and $x = 2$

slope of tangent @ $x = 1$



$M_{\text{sec}} = \dfrac{4 - 1}{2 - 1} = \boxed{3}$

$M_{\text{tan}} = \lim_{x \to 1} \dfrac{f(x) - f(1)}{x - 1} = \lim_{x \to 1} \dfrac{x^2 - 1}{x - 1}$

$$= \lim_{x\to 1} \frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}} = \lim_{x\to 1} x+1 = \boxed{2}$$

equation of secant line? pt $(1,1)$ $m=3$

$$\to \boxed{y-1 = 3(x-1)} \qquad \boxed{y-4 = 3(x-2)}$$

$$y-1 = 3x-3 \qquad y-4 = 3x-6$$

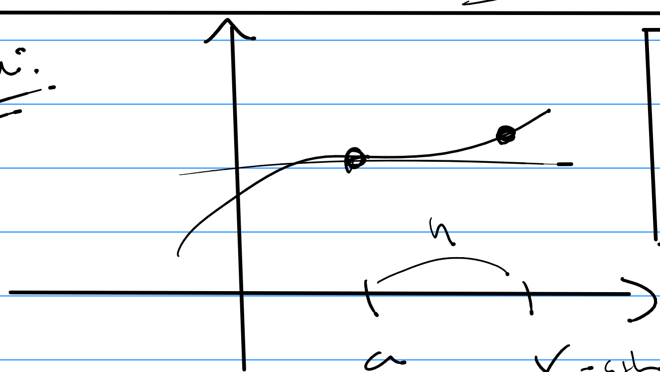
$$y = 3x-2 \qquad y = 3x-2$$

eqn of tangent line? pt $(1,1)$ $m=2$

$$\boxed{y-1 = 2(x-1)}$$

$$y = 2x-1$$

Notation:



$$\boxed{m_{tangent} = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}}$$

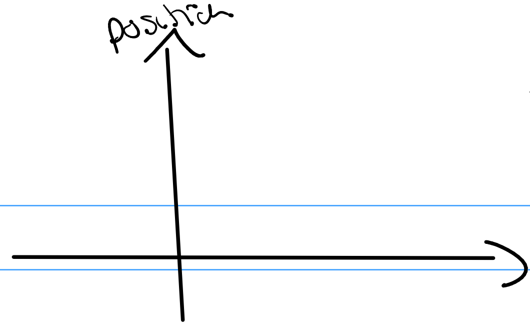
$x-a = h$

$x = a+h$

$\lim_{x\to a}$ same as $\lim_{h\to 0}$

$$\boxed{m_{tangent} = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}}$$

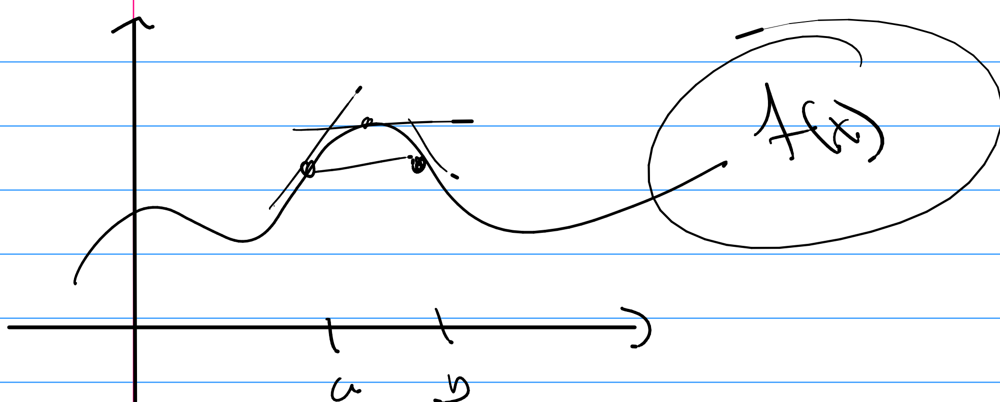
Application:



time

$$\text{slope} = \frac{\text{change in position}}{\text{change in time}}$$

$$(\text{velocity @ } x=a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$



$\boxed{\text{slope of tangent line}}$ of $f(x)$ @ $x=a$

$$m_{tan} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$f(x), a$

$\downarrow$

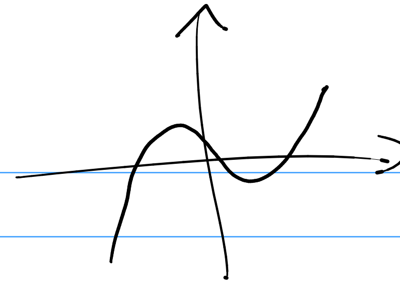
$$\boxed{\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$

operator

$\downarrow$

m_{tan}

ex $f(\square) = \square^3 - \square$



m_{tan} at $x = a$

$$m_{tan} = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h\to 0} \frac{[(a+h)^3 - (a+h)] - [a^3 - a]}{h}$$

$$\to m_{tan} = \lim_{h\to 0} \frac{\cancel{a^3} + 3a^2h + 3ah^2 + h^3 \cancel{- a} - h \cancel{- a^3} \cancel{+ a}}{h}$$

$$m_{tan} = \lim_{h\to 0} \frac{\cancel{h}(3a^2 + 3ah + h^2 - 1)}{\cancel{h}}$$

$$= 3a^2 + 3a(0) + (0)^2 - 1 = \boxed{3a^2 - 1}$$

b/c a is any number it's a variable.

$$\boxed{\text{slope @ } x \text{ for } f(x) \quad m_{tan} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}}$$

ex slope @ x for $f(x) = x^3 - x$

$m_{tan} = 3x^2 - 1$

(above)

So $\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ takes functions in and spits functions out.

it is an operator on functions...

$f(x)$

$\downarrow$

$$\boxed{\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

Derivative

Notation?

Derivative Notation

$D_x[f(x)]$

$\frac{d}{dx}[f(x)]$

$\frac{dy}{dx}$

$[f(x)]'$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

y'

$f(\square) = \sqrt{\square}$

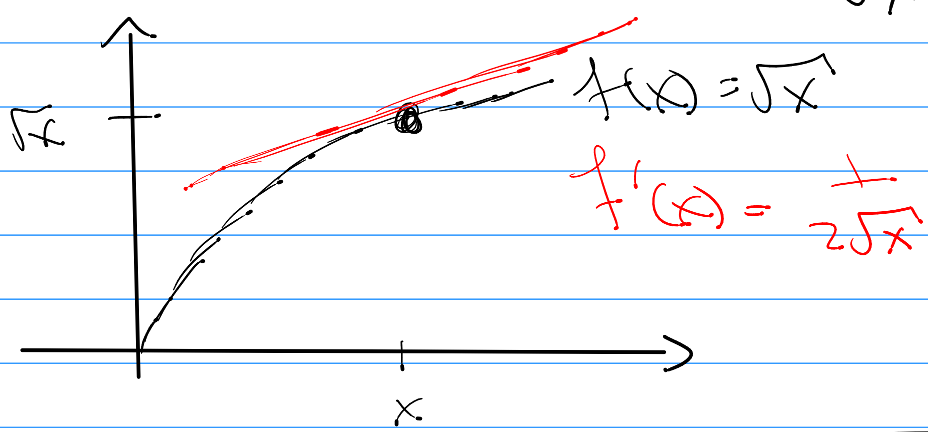
Ex $f(x) = \sqrt{x}$ find $f'(x)$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= \lim_{h\to 0} \frac{(\sqrt{x+h} - \sqrt{x})}{h} \frac{(\sqrt{x+h} + \sqrt{x})}{(\sqrt{x+h} + \sqrt{x})} = \lim_{h\to 0} \frac{(\cancel{x}+h) - \cancel{x}}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})} = \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$f(x) = \sqrt{x}$ $\qquad$ $f'(x) = \dfrac{1}{2\sqrt{x}}$



Goal? given $f(x) \to \boxed{\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}} \to f'(x)$