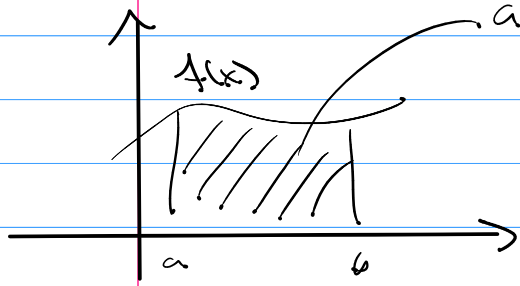
Math 242

Q5/ 4.1 #26



$$\text{area} = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

$$\Delta x = \frac{b-a}{n}$$

right endpts $x_i^* = a + i\Delta x$

#26 $f(x) = x^3$ over $[0, 1]$ $\Delta x = \frac{1-0}{n} = \frac{1}{n}$

$x_i = a + i\Delta x$: $x_1 = 0 + \frac{1}{n}$ $x_2 = 0 + \frac{2}{n}$ $x_3 = 0 + \frac{3}{n}$ .. $x_n = \frac{n}{n} = 1$

$x_1 = \frac{1}{n}, x_2 = \frac{2}{n}, x_3 = \frac{3}{n}, \dots, x_n = 1$

$x_i = i/n$

$$\text{area} = \lim_{n\to\infty} f(x_1)\Delta x + f(x_2)\Delta x + \dots + f(x_n)\Delta x$$

$$\boxed{\text{area} = \lim_{n\to\infty} \left(\frac{1}{n}\right)^3\left(\frac{1}{n}\right) + \left(\frac{2}{n}\right)^3\left(\frac{1}{n}\right) + \dots + (1)^3\left(\frac{1}{n}\right)}$$

as Σ-notation

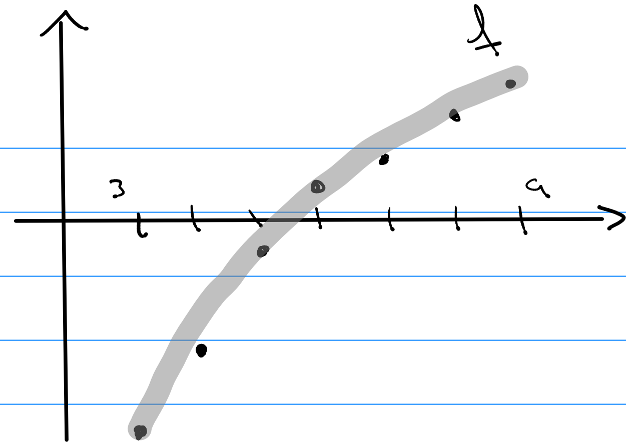
$$\text{area} = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^3\left(\frac{1}{n}\right)$$

$$\boxed{\text{area} = \lim_{n\to\infty} \frac{1}{n^4} \sum_{i=1}^{n} i^3} = \lim_{n\to+\infty} \frac{1}{n^4}\left(\frac{n(n+1)}{2}\right)^2$$

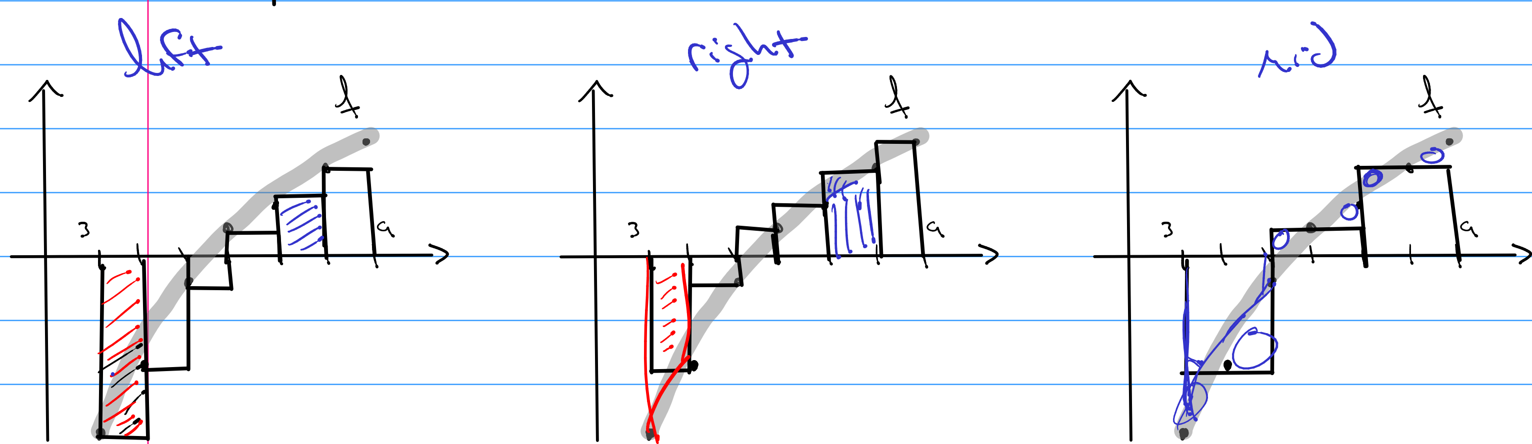
$$= \boxed{\frac{1}{4}}$$

4.2 #8

x	y = f(x)
3	-3.4
4	-2.1
5	-0.6
6	0.3
7	0.9
8	1.4
9	1.8

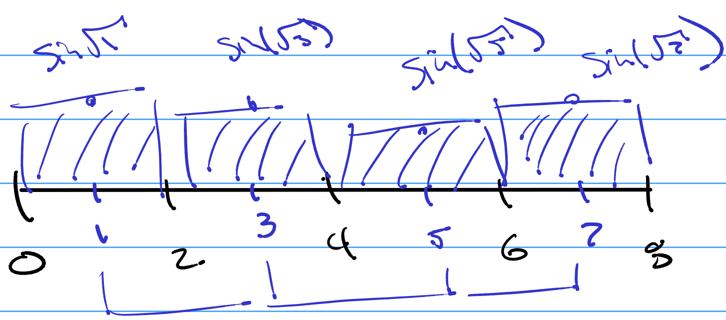


$$\int_3^9 f(x)\,dx$$



Sum rectangles $\approx$ Net signed area

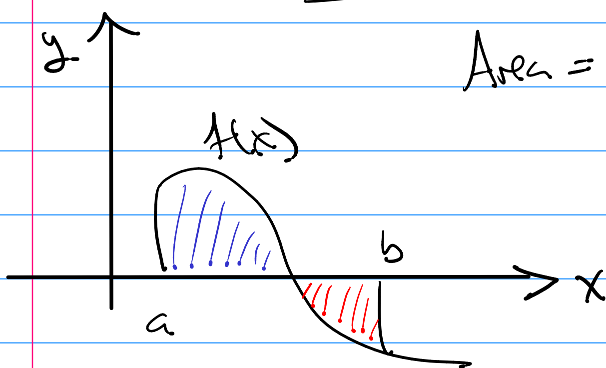
4.2 #9 $f(x) = \sin\sqrt{x}$ over $[0, 8]$ $n = 4$



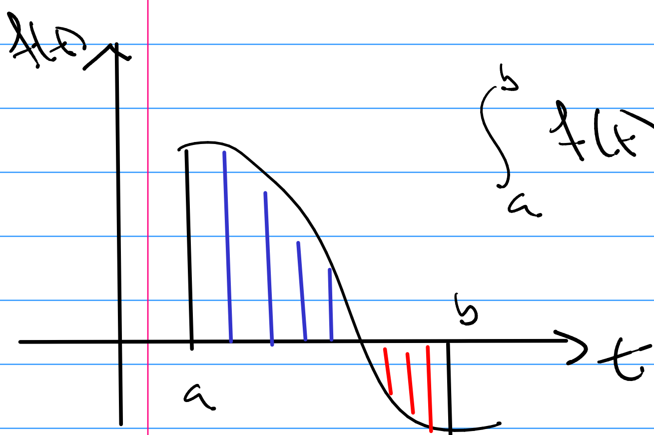
$$\Delta x = \frac{b-a}{n} = 2$$

$$\text{area} \approx \frac{\left(\sin(\sqrt{1}) + \sin(\sqrt{3}) + \sin(\sqrt{5}) + \sin(\sqrt{7})\right)}{4} \cdot 8 = 2$$

4.1/4.2 Area (Net signed area)



$$\text{Area} = \int_a^b f(x)\,dx = \lim_{\max \Delta x_i \to 0} \sum_{i=1}^{n} f(x_i^*)\Delta x_i$$

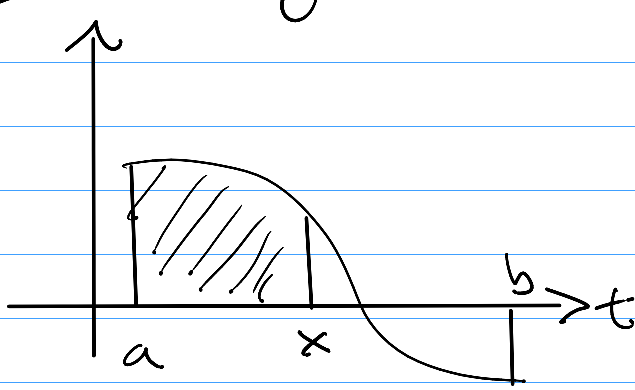


$$\int_a^b f(t)\,dt = \lim_{\max \Delta t_i \to 0} \sum_{i=1}^{n} f(t_i^*)\Delta t_i$$

Fundamental Theorem of Calculus

Parts:
1. do an area problem → antiderivatives
2. do an antiderivative problem → Area

Part 1 Net Signed Area accumulator



f is cont. on $[a,b]$

$$\underbrace{A(x) = \int_a^x f(t)\,dt}_{\text{area under } f \text{ over } [a,x]}$$

Properties of $A(x)$?

$$A(\square) = \int_a^{\square} f(t)\,dt$$

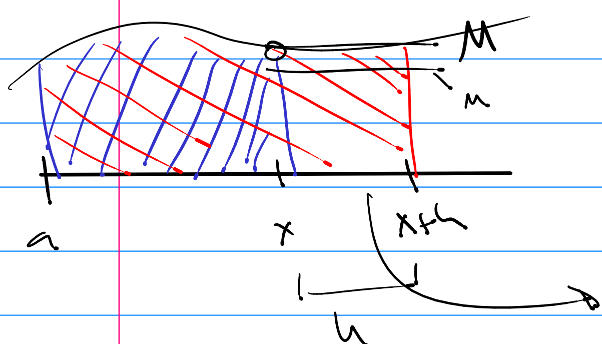
① $A(a) = \int_a^a f(t)\,dt = 0$

② $A(x)$ is a function ... what is $D_x[A(x)]$?

$$D_x[A(x)] = D_x\left[\int_a^x f(t)\,dt\right] = ?$$

$$D_x[A(x)] = \lim_{h\to 0} \frac{A(x+h) - A(x)}{h}$$

So $D_x[A(x)] = \lim_{h\to 0} \dfrac{\int_a^{x+h} f(t)\,dt - \int_a^x f(t)\,dt}{h}$



$$= \lim_{h\to 0} \frac{\int_x^{x+h} f(t)\,dt}{h}$$

$$m \le f(t) \le M$$

$$mh \le \int_x^{x+h} f(t)\,dt \le Mh$$

$$m \le \frac{\int_x^{x+h} f(t)\,dt}{h} \le M$$

as $h\to 0$ $m \to f(x)$

$h\to 0$ $M \to f(x)$

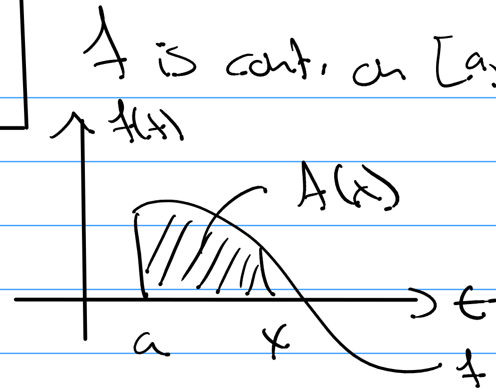
So $D_x[A(x)] = \lim_{h\to 0} \dfrac{\int_x^{x+h} f(t)\,dt}{h} = f(x)$

b/c $D_x[A(x)] = f(x)$

says $A(x)$ is an antiderivative of $f(x)$

Fundamental thm of Calculus part 1

f is cont. on $[a,b]$



let $A(x) = \int_a^x f(t)\,dt$

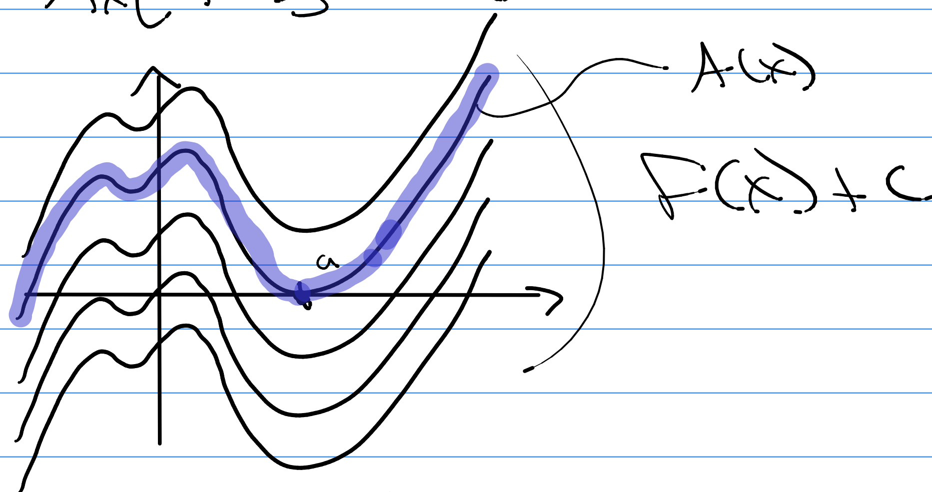
(1) $A(x)$ is an antiderivative of $f(x)$ w/C

$$D_x[A(x)] = f(x)$$

(2) $A(a) = 0$

How does $A(x)$ relate to $A_x[f(x)]$?

ex $A_x[f(x)] = F(x) + C$



Use?

$A(x) = \int_a^x f(t)\,dt$ is a way to "find" antiderivatives that can't be done by normal ways.

→ $f(x) = \sin(x)$ $\quad A_x[\sin x] = -\cos x + C$

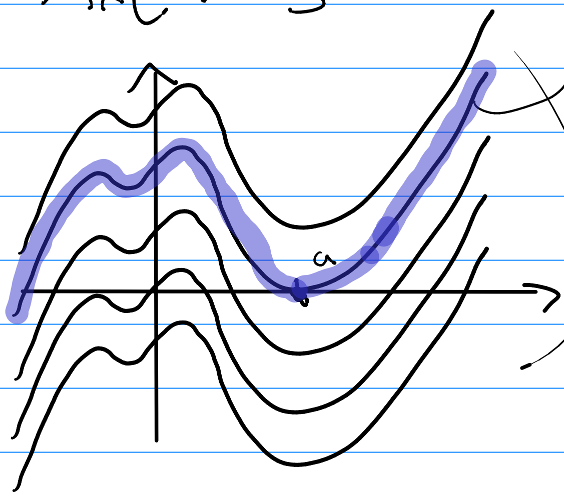
$f(x) = e^{-x^2}$ $\quad A_x[e^{-x^2}] = ?$

$A(x) = \int_0^x e^{-t^2}\,dt$ is <u>an</u> antideriv of e^{-x^2}

$A(4) = \int_0^4 e^{-t^2}\,dt$

Part 2

ex $A_x[f(x)] = F(x) + C$



$A(x) = \int_a^x f(t)\,dt$

$F(x) + C$

Says $F(x) = A(x) + C$

Consider: $F(b) - F(a)$

$= (A(b) + c) - (A(a) + c)$

$= A(b) - A(a)$

$= \int_a^b f(t)\,dt - \int_a^a f(t)\,dt$

$= \int_a^b f(t)\,dt$ = area under f over $[a,b]$

$\boxed{\text{Fund. thm of Calculus part 2}}$ f is cont on $[a,b]$

$$A_x[f(x)] = F(x) \quad \text{let } c=0$$

$$\text{then } \int_a^b f(x)\,dx = F(b) - F(a)$$

ex Area under $f(x)=x^3$ over $[0,1]$

$$\int_0^1 x^3\,dx = \frac{1}{4}(1)^4 - \frac{1}{4}(0)^4 = \boxed{\frac{1}{4}}$$

$$A_x[x^3] = \frac{1}{4}x^4 + c$$

$\boxed{\text{Fund. thm of Calculus}}$ $f(x)$ is cont on $[a,b]$

1. (use areas to create antiderivatives)

$$A(x) = \int_a^x f(t)\,dt$$

<u>then:</u> a) $D_x[A(x)] = f(x)$

b) $A(a) = 0$

2. (use antideriv to find areas)

☆ $A_x[f(x)] = F(x) + C$ (let $c=0$)

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Using part 2 $\quad \int_a^b f(x)\,dx$ = Area under f over $[a,b]$

$$\int_a^b f(x)\,dx = ?$$

① $A_x[f(x)] = F(x) + C$

② $\int_a^b f(x)\,dx = F(b) - F(a)$

Notation:

① Indefinite Integral Notation..

$$A_x[f(x)] = \int f(x)\,dx = F(x) + C$$

Now:

$$\int_a^b f(x)\,dx = \Big[\underbrace{\int f(x)\,dx}_{A_x[f(x)]}\Big]\Big|_{x=a}^{x=b} = \big[F(x)\big]\Big|_{x=a}^{x=b}$$

$$= F(b) - F(a)$$

Ex $\quad \int_1^3 (x^{-4/3} + 1)\,dx = \left[\int (x^{-4/3} + 1)\,dx\right]_{x=1}^{x=3}$

$$= \left[3x^{1/3} + x\right]\Big|_{x=1}^{x=3} = \left(3(3)^{1/3} + 3\right) - \left(3(1)^{1/3} + 1\right)$$

$$= \boxed{3^{4/3} - 1}$$