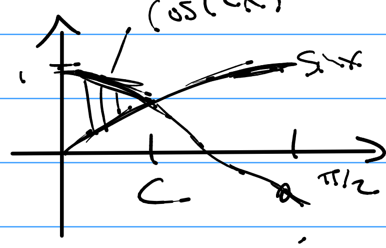
Math 242

Q's

5.1 #35 $\int_0^{\pi/2} |\sin x - \cos 2x|\, dx = \text{Area}$

area between $\sin(x)$, $\cos(2x)$



$$\text{Area} = \int_0^{c} (\cos(2x) - \sin x)\, dx + \int_c^{\pi/2} (\sin(x) - \cos(2x))\, dx$$

Cross?

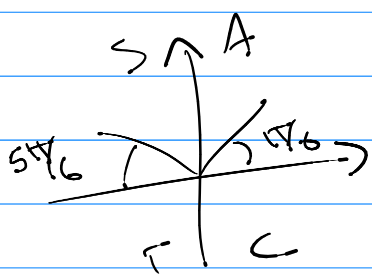
$$\sin x = \cos(2x)$$
$$\sin x = 1 - 2\sin^2 x$$
$$2\sin^2 x + \sin x - 1 = 0$$

$$(2\sin x - 1)(\sin x + 1) = 0$$

$$\sin x = \tfrac{1}{2} \qquad \sin x = -1$$

$$x = \boxed{\pi/6} + 2n\pi \qquad x = 3\pi/2 + 2n\pi$$

$$x = 5\pi/6 + 2n\pi$$



$$\text{Area} = \int_0^{\pi/6} (\cos 2x - \sin x)\, dx + \int_{\pi/6}^{\pi/2} (\sin x - \cos 2x)\, dx$$

Note: $\int \sin x\, dx = -\cos x + C$

Note: $\int \cos(2x)\,dx = \frac{1}{2}\int \cos u\,du = \frac{1}{2}\sin u + C$

$= \frac{1}{2}\sin(2x) + C$

$u = 2x$

$du = 2\,dx$

Continue

$$\text{Area} = \int_0^{\pi/6} (\cos(2x) - \sin x)\,dx + \int_{\pi/6}^{\pi/2} (\sin x - \cos(2x))\,dx$$

$$= \left[\frac{1}{2}\sin(2x) + \cos x\right]\Bigg|_{x=0}^{x=\pi/6} + \left[-\cos x - \frac{1}{2}\sin(2x)\right]\Bigg|_{x=\pi/6}^{x=\pi/2}$$

$$= \left[\left(\frac{1}{2}\sin(\pi/3) + \cos(\pi/6)\right) - (0+1)\right] + \left[(0-0) - \left(-\cos(\pi/6) - \frac{1}{2}\sin(\pi/3)\right)\right]$$

$$= \boxed{2\cos(\pi/6) + \sin(\pi/3) - 1}$$

Applications of ① $\int f(x)\,dx = F(x) + C$

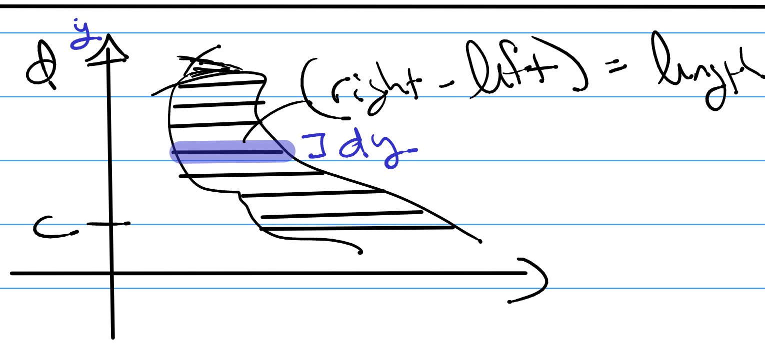
② $\int_a^b f(x)\,dx =$ net signed area

---

① Areas between curves

$$\text{Area} = \int_c^d (\text{length}) \cdot dy$$

function of y



$$\text{Area} = \int_a^b \boxed{\text{Area's}} = \int_a^b (\text{length})(dx)$$

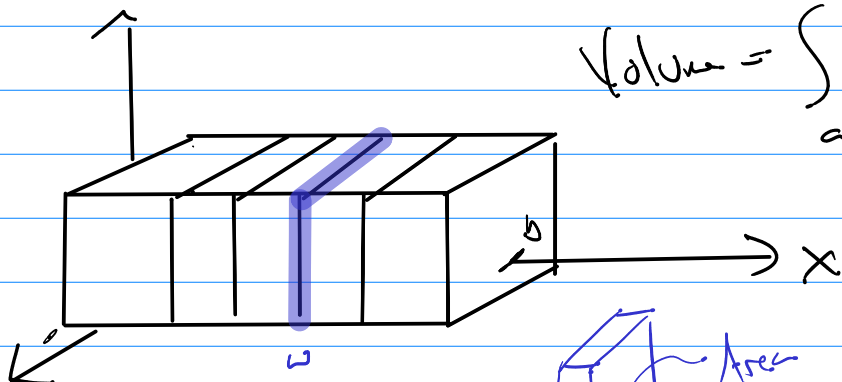
(length)(dx)

slice in x direction from a to b

function of x

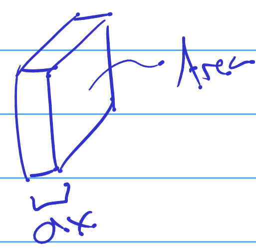
slices in x-direction

Can we also find Volumes? (slice)



$$\text{Volume} = \int_a^b (\text{Area})\,dx$$

function of x
$A(x)$



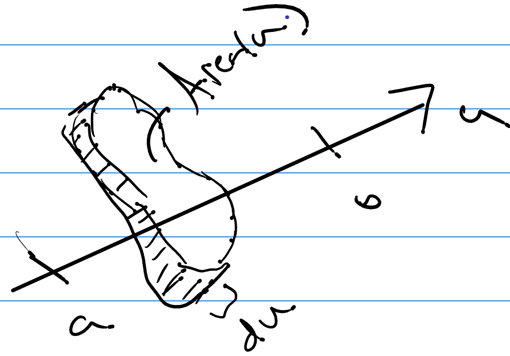
$\to V = (\text{Area})\,dx$

So

Area by slicing

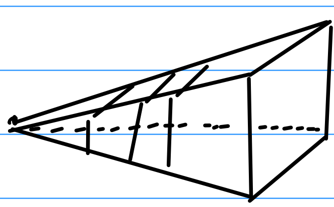
$$\int_a^b \text{Area}(u)\,du$$

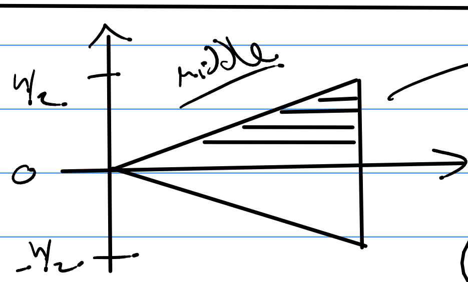
Slicing in u-direction from u=a to u=b

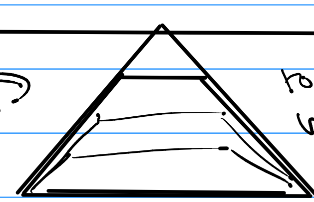


---

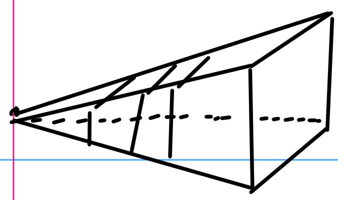
ex

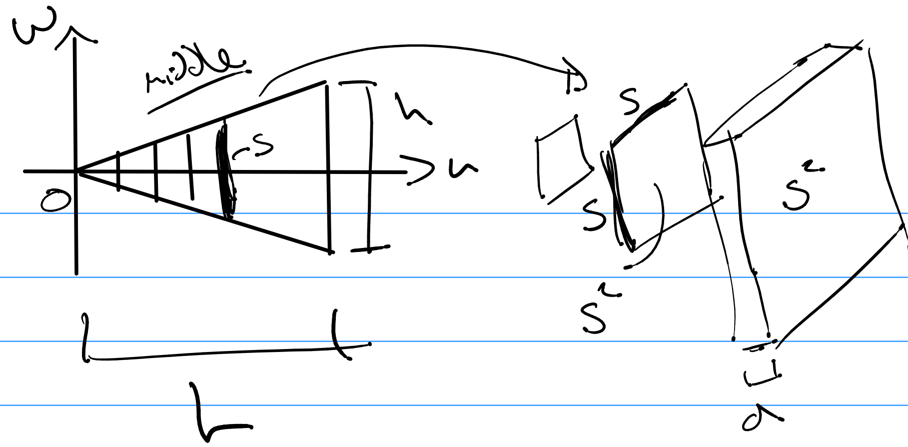






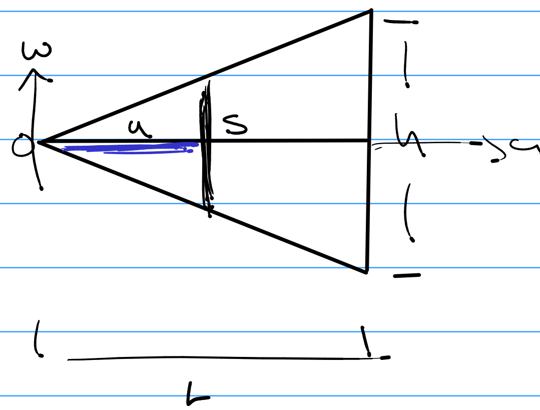
bad?





$$V=\int_0^L \text{Area}(u)\,du$$

s^2



$$\frac{L}{h}=\frac{u}{s}$$

$$\boxed{s=\frac{h}{L}u}$$

$u=0 \quad s=0$

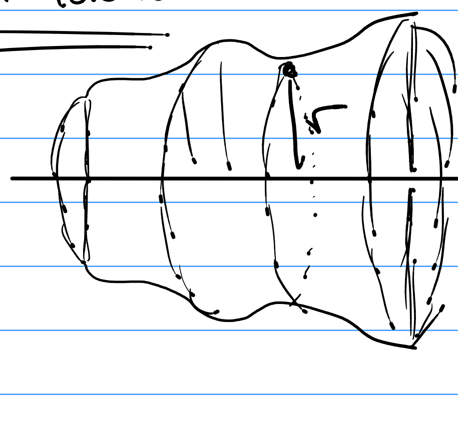
$u=L \quad s=h$

$$V=\int_0^L\left(\frac{h}{L}u\right)^2du=\frac{h^2}{L^2}\int_0^L u^2\,du$$

$$=\frac{h^2}{3L^2}u^3\Big|_0^h=\boxed{\frac{1}{3}h^2L}$$

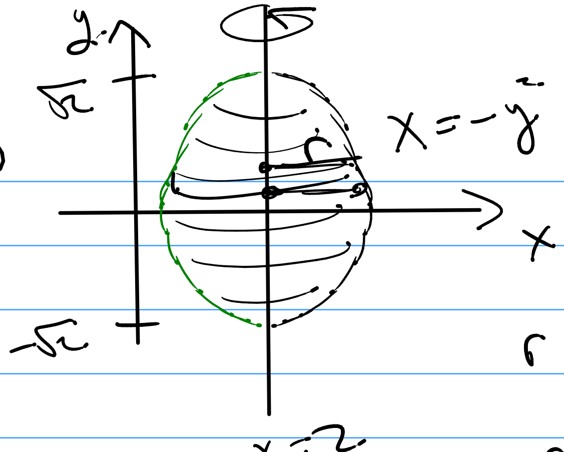
Solids of Revolution → slice along / perpendicular axis of revolution = circle

$= \pi r^2$



axis of revolution

Ex



$x=-y^2+4$

$V = 2\int_0^{\sqrt{2}} \underbrace{Area(y)}_{\pi r^2}\, dy$

$r = (-y^2+4)-(2)$

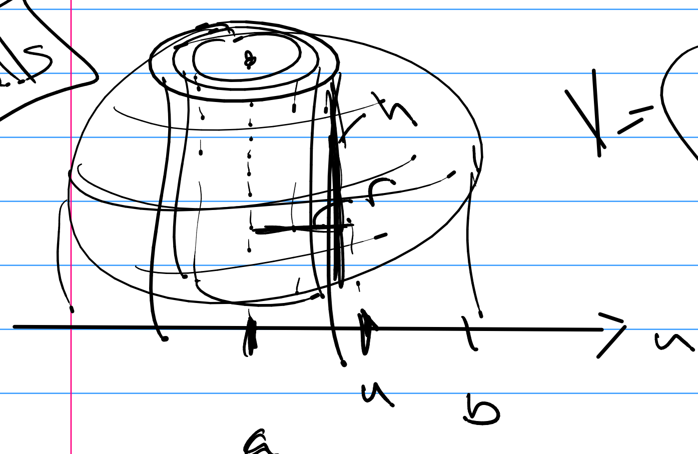
$r = (-y^2+2)$

$V = 2\int_0^{\sqrt{2}} \pi(-y^2+2)^2\, dy$

$V = 2\pi\int_0^{\sqrt{2}} (-y^2+2)^2\, dy = 2\pi\int_0^{\sqrt{2}} (y^4-4y^2+4)\, dy$

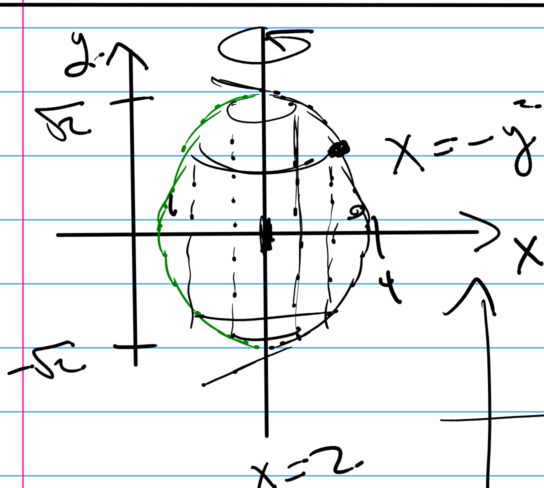
$= \underline{etc}$

Shells



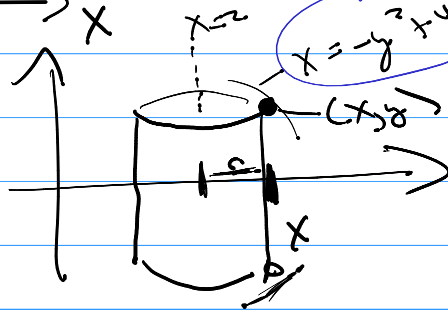
$V = \int_a^b \underline{Area} \cdot du$

Ex 2



$x=-y^2+4$

$V = \int^4 \underbrace{2\pi rh}_{\text{function of } x}\, dx$



$x=-y^2+4$

$h = 2y = 2(4-x)^{1/2}$

$r = x-2$

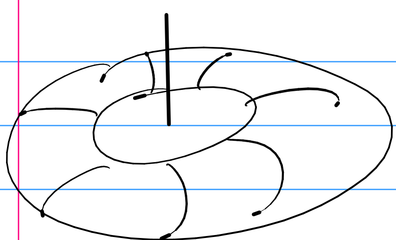
$$V = \int_2^4 2\pi (x-2)\left(2\sqrt{4-x}\right) dx$$

$$V = 4\pi \int_2^4 (x-2)\sqrt{4-x}\, dx = 4\pi \int_0^2 (2-u)u^{1/2}\, du$$

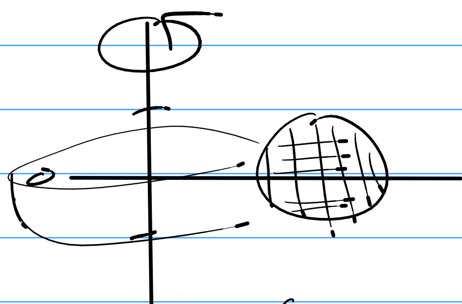
let $u = 4-x \Rightarrow x-2 = 2-u$

$du = -dx$

$=$ finish



Volume of donut?



$\pi r_2^2 - \pi r_1^2$

(see video)

(Do for fun @ home)